The Discriminant of a Quadratic Function

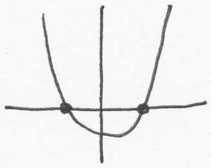
The discriminant is the name given to the expression that appears under the square root sign in the quadratic formula:

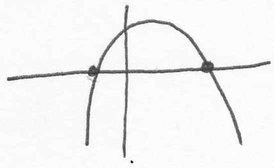
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The discriminant is given by $b^2 - 4ac$. This tells you about the 'nature' of the roots of a quadratic equation.

NB - The ***roots*** of a quadratic equation = the ***solutions*** of a quadratic equation = where the graph of the quadratic ***crosses the x-axis***.

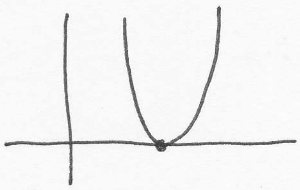
If the value of the discriminant is positive, ie $b^2 - 4ac > 0$, then there will be two real roots and the graph will look like:

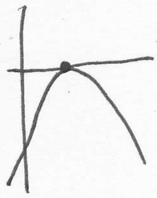




graph crosses x-axis twice

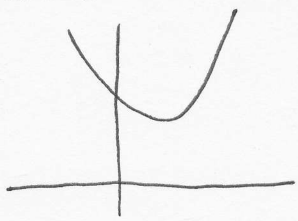
If the value of the discriminant is zero, ie $b^2 - 4ac = 0$, then there will be one real root (also called repeated roots) and the graph will look like:

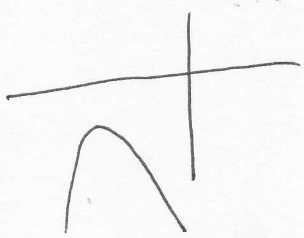




graph touches the x-axis

If the value of the discriminant is negative, ie $b^2 - 4ac < 0$, then there will be no real roots (the roots are complex – more if you do A Level Further Maths!) and the graph will look like:





graph doesn't touch/cross x axis.

Examples

Determine the nature of the roots of the following quadratic functions and sketch their graphs:

1. $y = x^2 + 6x + 5$
2. $y = x^2 - 2x + 1$
3. $y = x^2 - 3x + 10$
4. $y = -x^2 + 4x - 5$
5. Find the ~~range of~~ values of k for which $x^2 + kx + 3 = 0$ has one ~~two distinct~~ real root~~s~~.

Eg1 $y = x^2 + 6x + 5$

$b^2 - 4ac$ $\quad 6^2 - 4 \times 1 \times 5 = 36 - 20 = 16 > 0$ $\therefore$ two real roots

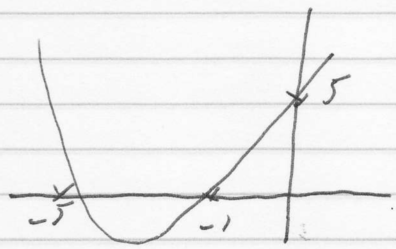
crosses y axis when $x = 0$ $\quad y = 0 + 0 + 5 = 5$ $\quad (0,5)$

crosses x axis when $y = 0$ $\quad x^2 + 6x + 5 = 0$

$(x+5)(x+1) = 0$

$x = -1, \; x = -5$ $\quad (-1,0)$ & $(-5,0)$

x^2 is +ve $\therefore$ U



eg2 $y = x^2 - 2x + 1$

$b^2 - 4ac$ $\quad (-2)^2 - 4 \times 1 \times 1 = 4 - 4 = 0$ $\therefore$ one root

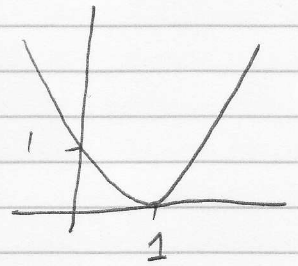
crosses y axis when $x = 0$ $\quad y = 1$ $\quad (0,1)$

crosses x axis when $y = 0$ $\quad x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$x = 1$

x^2 is +ve $\therefore$ U



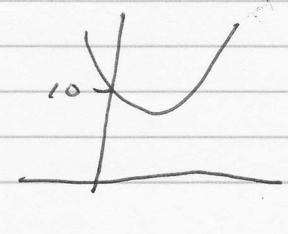
eg3 $y = x^2 - 3x + 10$

$b^2 - 4ac$: $\quad (-3)^2 - 4 \times 1 \times 10 = 9 - 40 = -31 < 0$ $\therefore$ no real roots

crosses y axis when $x = 0$ $\quad y = 10$ $\quad (0,10)$

doesn't cross x axis

x^2 +ve $\therefore$ U

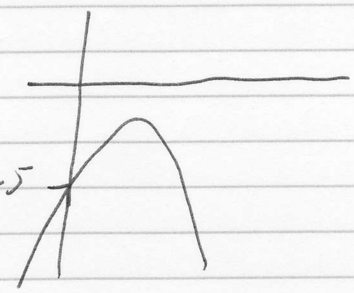


eg4 $y = -x^2 + 4x - 5$

$b^2 - 4ac \qquad 4^2 - 4 \times -1 \times -5 = 16 - 20 = -4 < 0 \therefore$ no real roots

Crosses y axis when $x = 0 \quad y = -5 \qquad (0, -5)$

$-ve\ x^2 \therefore \cap$



eg5 $x^2 + kx + 3$

for two roots $\quad b^2 - 4ac \geq 0$

$k^2 - 4 \times 1 \times 3 \geq 0$

$k^2 - 12 \geq 0$

$k^2 \geq 12$

$k \geq \pm\sqrt{12}$

$k \geq \pm 2\sqrt{3}$

$\therefore \cancel{|k| > 2\sqrt{3} \text{ or } k < -2\sqrt{3}}$