Leave blank

10. The line l_1 passes through the point A (2, 5) and has gradient $-\frac{1}{2}$.

(a) Find an equation of l_1, giving your answer in the form $y = mx + c$.

(3)

The point B has coordinates (–2, 7).

(b) Show that B lies on l_1.

(1)

(c) Find the length of AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.

(3)

The point C lies on l_1 and has x-coordinate equal to p.

The length of AC is 5 units.

(d) Show that p satisfies

$$p^2 - 4p - 16 = 0.$$

(4)

Leave blank

11. The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \qquad x > 0.$$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$.
(6)

(b) Find an equation of the normal to C at the point P.
(3)

The tangent at P meets the x-axis at A and the normal at P meets the x-axis at B.

(c) Find the area of triangle APB.
(4)

Leave blank

5. The curve C has equation $y = x(5 - x)$ and the line L has equation $2y = 5x + 4$

(a) Use algebra to show that C and L do not intersect.

(4)

(b) In the space on page 11, sketch C and L on the same diagram, showing the coordinates of the points at which C and L meet the axes.

(4)

Leave blank

6.

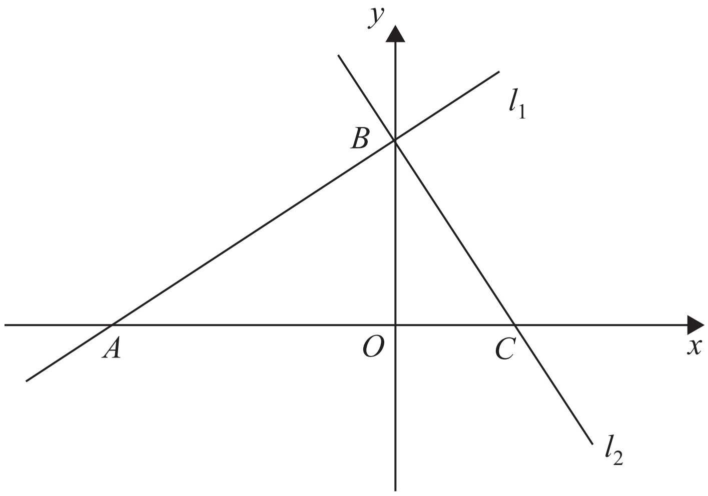


Figure 1

The line l_1 has equation $2x - 3y + 12 = 0$

(a) Find the gradient of l_1.

(1)

The line l_1 crosses the x-axis at the point A and the y-axis at the point B, as shown in Figure 1.

The line l_2 is perpendicular to l_1 and passes through B.

(b) Find an equation of l_2.

(3)

The line l_2 crosses the x-axis at the point C.

(c) Find the area of triangle ABC.

(4)

Leave blank

8. The curve C_1 has equation

$$y = x^2(x+2)$$

(a) Find $\frac{dy}{dx}$

(2)

(b) Sketch C_1, showing the coordinates of the points where C_1 meets the x-axis.

(3)

(c) Find the gradient of C_1 at each point where C_1 meets the x-axis.

(2)

The curve C_2 has equation

$$y = (x-k)^2(x-k+2)$$

where k is a constant and $k > 2$

(d) Sketch C_2, showing the coordinates of the points where C_2 meets the x and y axes.

(3)

Leave blank

7. The curve C has equation $y = f(x)$, $x \neq 0$, and the point $P(2, 1)$ lies on C. Given that

$$f'(x) = 3x^2 - 6 - \frac{8}{x^2},$$

(a) find f(x).

(5)

(b) Find an equation for the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers.

(4)

Leave blank

8. The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2, \quad x > 0$.

(a) Find an expression for $\dfrac{dy}{dx}$.

(3)

(b) Show that the point P (4, 8) lies on C.

(1)

(c) Show that an equation of the normal to C at the point P is

$$3y = x + 20.$$

(4)

The normal to C at P cuts the x-axis at the point Q.

(d) Find the length PQ, giving your answer in a simplified surd form.

(3)

Leave blank

3. The points P and Q have coordinates $(-1, 6)$ and $(9, 0)$ respectively.

The line l is perpendicular to PQ and passes through the mid-point of PQ.

Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

Leave blank

10. The curve C has equation

$$y = (x+1)(x+3)^2$$

(a) Sketch C, showing the coordinates of the points at which C meets the axes.

(4)

(b) Show that $\frac{dy}{dx} = 3x^2 + 14x + 15$.

(3)

The point A, with x-coordinate -5, lies on C.

(c) Find the equation of the tangent to C at A, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Another point B also lies on C. The tangents to C at A and B are parallel.

(d) Find the x-coordinate of B.

(3)

Leave blank

10. The curve C has equation $y = x^2(x-6) + \frac{4}{x}$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 2 respectively.

(a) Show that the length of PQ is $\sqrt{170}$.

(4)

(b) Show that the tangents to C at P and Q are parallel.

(5)

(c) Find an equation for the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Leave blank

11. The line l_1 has equation $y = 3x + 2$ and the line l_2 has equation $3x + 2y - 8 = 0$.

(a) Find the gradient of the line l_2.

(2)

The point of intersection of l_1 and l_2 is P.

(b) Find the coordinates of P.

(3)

The lines l_1 and l_2 cross the line $y = 1$ at the points A and B respectively.

(c) Find the area of triangle ABP.

(4)

Leave blank

10. The curve C with equation $y = f(x)$, $x \neq 0$, passes through the point $(3, 7\frac{1}{2})$.

Given that $f'(x) = 2x + \frac{3}{x^2}$,

(a) find $f(x)$.

(5)

(b) Verify that $f(-2) = 5$.

(1)

(c) Find an equation for the tangent to C at the point $(-2, 5)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Leave blank

11. The line l_1 passes through the points $P(-1, 2)$ and $Q(11, 8)$.

(a) Find an equation for l_1 in the form $y = mx + c$, where m and c are constants.

(4)

The line l_2 passes through the point $R(10, 0)$ and is perpendicular to l_1. The lines l_1 and l_2 intersect at the point S.

(b) Calculate the coordinates of S.

(5)

(c) Show that the length of RS is $3\sqrt{5}$.

(2)

(d) Hence, or otherwise, find the exact area of triangle PQR.

(4)

Leave blank

10.

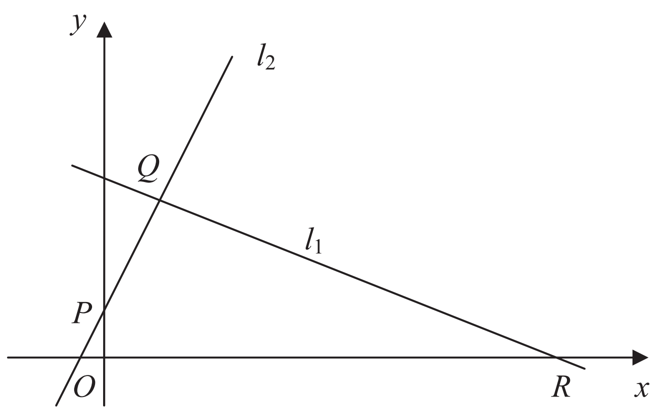


Figure 2

The points Q (1, 3) and R (7, 0) lie on the line l_1, as shown in Figure 2.

The length of QR is $a\sqrt{5}$.

(a) Find the value of a.

(3)

The line l_2 is perpendicular to l_1, passes through Q and crosses the y-axis at the point P, as shown in Figure 2.

Find

(b) an equation for l_2,

(5)

(c) the coordinates of P,

(1)

(d) the area of ΔPQR.

(4)

Leave blank

4. The point A (–6, 4) and the point B (8, –3) lie on the line L.

(a) Find an equation for L in the form $ax + by + c = 0$, where a, b and c are integers.
(4)

(b) Find the distance AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.
(3)

Q4

(Total 7 marks)

Leave blank

9. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, and $\mathrm{f}'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2}$.

Given that the point $P(4, 1)$ lies on C,

(a) find $\mathrm{f}(x)$ and simplify your answer.

(6)

(b) Find an equation of the normal to C at the point $P(4, 1)$.

(4)

Leave blank

8. The point P (1, a) lies on the curve with equation $y = (x + 1)^2(2 - x)$.

(a) Find the value of a.

(1)

(b) On the axes below sketch the curves with the following equations:

(i) $y = (x + 1)^2(2 - x)$,

(ii) $y = \frac{2}{x}$.

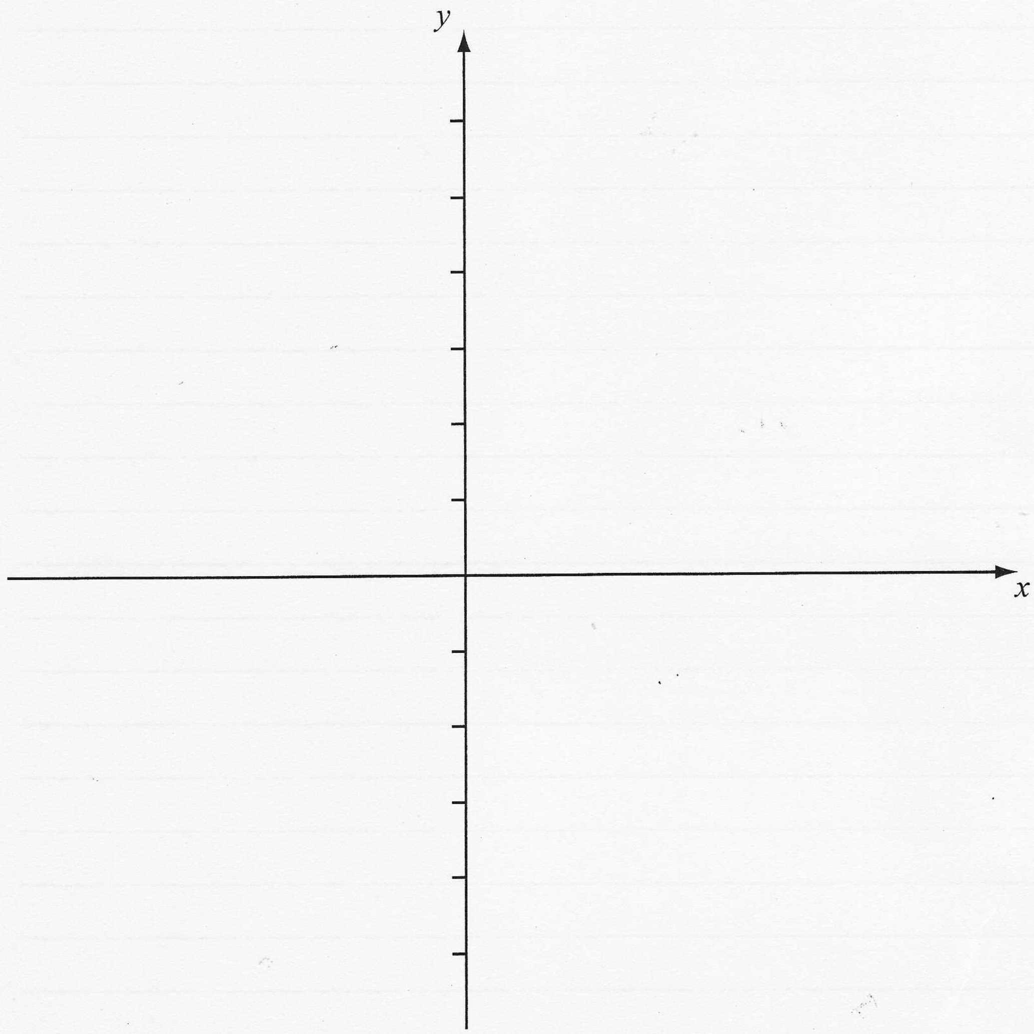
On your diagram show clearly the coordinates of any points at which the curves meet the axes.

(5)

(c) With reference to your diagram in part (b) state the number of real solutions to the equation

$$(x+1)^2(2-x) = \frac{2}{x}.$$

(1)



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9. The curve C with equation $y = \mathrm{f}(x)$ passes through the point (5, 65).

Given that $\mathrm{f}'(x) = 6x^2 - 10x - 12$,

(a) use integration to find $\mathrm{f}(x)$.

(4)

(b) Hence show that $\mathrm{f}(x) = x(2x+3)(x-4)$.

(2)

(c) In the space provided on page 17, sketch C, showing the coordinates of the points where C crosses the x-axis.

(3)