C3

Chapter 3

Exponential and log functions

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5. The radioactive decay of a substance is given by

$$R = 1000\mathrm{e}^{-ct}, \qquad t \geqslant 0.$$

where R is the number of atoms at time t years and c is a positive constant.

(a) Find the number of atoms when the substance started to decay.

(1)

It takes 5730 years for half of the substance to decay.

(b) Find the value of c to 3 significant figures.

(4)

(c) Calculate the number of atoms that will be left when $t = 22\ 920$.

(2)

(d) In the space provided on page 13, sketch the graph of R against t .

(2)

Leave blank

9. (i) Find the exact solutions to the equations

(a) $\ln(3x - 7) = 5$

(3)

(b) $3^x e^{7x+2} = 15$

(5)

(ii) The functions f and g are defined by

$$f(x) = e^{2x} + 3, \qquad x \in \mathbb{R}$$
$$g(x) = \ln(x - 1), \qquad x \in \mathbb{R},\ x > 1$$

(a) Find f^{-1} and state its domain.

(4)

(b) Find fg and state its range.

(3)

Leave blank

1. The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y-coordinate of P is 8.

(a) Find, in terms of ln 2, the x-coordinate of P.

(2)

(b) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found.

(4)

Leave blank

3. The area, A mm^2, of a bacterial culture growing in milk, t hours after midday, is given by

$$A = 20\,e^{1.5t}, \quad t \geqslant 0$$

(a) Write down the area of the culture at midday.

(1)

(b) Find the time at which the area of the culture is twice its area at midday. Give your answer to the nearest minute.

(5)

Leave blank

1. Find the exact solutions to the equations

(a) $\ln x + \ln 3 = \ln 6$,

(2)

(b) $e^{x} + 3e^{-x} = 4$.

(4)

Leave blank

8. The amount of a certain type of drug in the bloodstream t hours after it has been taken is given by the formula

$$x = D\mathrm{e}^{-\frac{1}{8}t},$$

where x is the amount of the drug in the bloodstream in milligrams and D is the dose given in milligrams.

A dose of 10 mg of the drug is given.

(a) Find the amount of the drug in the bloodstream 5 hours after the dose is given. Give your answer in mg to 3 decimal places.

(2)

A second dose of 10 mg is given after 5 hours.

(b) Show that the amount of the drug in the bloodstream 1 hour after the second dose is 13.549 mg to 3 decimal places.

(2)

No more doses of the drug are given. At time T hours after the second dose is given, the amount of the drug in the bloodstream is 3 mg.

(c) Find the value of T.

(3)

Leave blank

3. Rabbits were introduced onto an island. The number of rabbits, P, t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \qquad t \in \mathbb{R},\ t \geqslant 0$$

(a) Write down the number of rabbits that were introduced to the island.

(1)

(b) Find the number of years it would take for the number of rabbits to first exceed 1000.

(2)

(c) Find $\dfrac{dP}{dt}$.

(2)

(d) Find P when $\dfrac{dP}{dt} = 50$.

(3)

Leave blank

5. The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = p\,e^{-kt}$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Write down the value of p.

(1)

(b) Show that $k = \frac{1}{4}\ln 3$.

(4)

(c) Find the value of t when $\frac{dm}{dt} = -0.6\ln 3$.

(6)

Leave blank

4. Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, θ°C, of the tea is modelled by the equation

$$\theta = 20 + Ae^{-kt},$$

where A and k are positive constants.

Given that the initial temperature of the tea was 90°C,

(a) find the value of A.

(2)

The tea takes 5 minutes to decrease in temperature from 90°C to 55°C.

(b) Show that $k = \frac{1}{5}\ln 2$.

(3)

(c) Find the rate at which the temperature of the tea is decreasing at the instant when $t = 10$. Give your answer, in °C per minute, to 3 decimal places.

(3)