# C3

# Chapter 2

# Functions

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**8.** The functions f and g are defined by

$$\mathrm{f} : x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}$$

$$\mathrm{g} : x \mapsto \frac{3}{x} - 4, \quad x > 0, \quad x \in \mathbb{R}$$

(a) Find the inverse function $\mathrm{f}^{-1}$.

**(2)**

(b) Show that the composite function gf is

$$\mathrm{gf} : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

**(4)**

(c) Solve $\mathrm{gf}(x) = 0$.

**(2)**

(d) Use calculus to find the coordinates of the stationary point on the graph of $y = \mathrm{gf}(x)$.

**(5)**



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**5.** The functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R}$$
$$\mathrm{g} : x \mapsto \mathrm{e}^{x^2}, \quad x \in \mathbb{R}$$

(a) Write down the range of g.

**(1)**

(b) Show that the composite function fg is defined by

$$\mathrm{fg} : x \mapsto x^2 + 3\mathrm{e}^{x^2}, \quad x \in \mathbb{R}.$$

**(2)**

(c) Write down the range of fg.

**(1)**

(d) Solve the equation $\frac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{fg}(x)\right] = x(x\mathrm{e}^{x^2} + 2)$.

**(6)**



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**6.** The function f is defined by

$$\mathrm{f}: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R},\ x \neq 5$$

(a) Find $\mathrm{f}^{-1}(x)$.

**(3)**

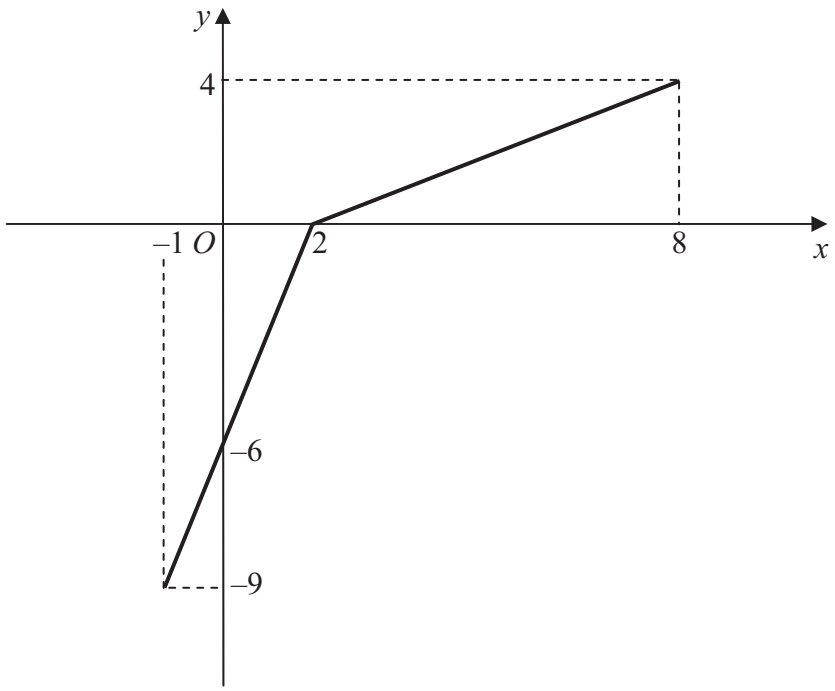


**Figure 2**

The function g has domain $-1 \leqslant x \leqslant 8$, and is linear from $(-1, -9)$ to $(2, 0)$ and from $(2, 0)$ to $(8, 4)$. Figure 2 shows a sketch of the graph of $y = \mathrm{g}(x)$.

(b) Write down the range of g.

**(1)**

(c) Find gg(2).

**(2)**

(d) Find fg(8).

**(2)**

(e) On separate diagrams, sketch the graph with equation

(i) $y = |\mathrm{g}(x)|$,

(ii) $y = \mathrm{g}^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

**(4)**

(f) State the domain of the inverse function $\mathrm{g}^{-1}$.

**(1)**



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**4.** The function f is defined by

$$\mathrm{f} : x \mapsto 4 - \ln(x+2), \quad x \in \mathbb{R}, \ x \geqslant -1$$

(a) Find $\mathrm{f}^{-1}(x)$.

**(3)**

(b) Find the domain of $\mathrm{f}^{-1}$.

**(1)**

The function g is defined by

$$\mathrm{g} : x \mapsto \mathrm{e}^{x^2} - 2, \quad x \in \mathbb{R}$$

(c) Find $\mathrm{fg}(x)$, giving your answer in its simplest form.

**(3)**

(d) Find the range of fg.

**(1)**



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**4.** The function f is defined by

$$\mathrm{f} : x \mapsto |2x - 5|, \quad x \in \mathbb{R}$$

(a) Sketch the graph with equation $y = \mathrm{f}(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

**(2)**

(b) Solve $\mathrm{f}(x) = 15 + x$.

**(3)**

The function g is defined by

$$\mathrm{g} : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leqslant x \leqslant 5$$

(c) Find $\mathrm{fg}(2)$.

**(2)**

(d) Find the range of g.

**(3)**



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**5.** The functions f and g are defined by

$$\mathrm{f} : x \mapsto \ln(2x-1), \qquad x \in \mathbb{R},\ x > \frac{1}{2},$$

$$\mathrm{g} : x \mapsto \frac{2}{x-3}, \qquad x \in \mathbb{R},\ x \neq 3.$$

(a) Find the exact value of fg(4).

**(2)**

(b) Find the inverse function $\mathrm{f}^{-1}(x)$, stating its domain.

**(4)**

(c) Sketch the graph of $y = |\,\mathrm{g}(x)\,|$. Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the $y$-axis.

**(3)**

(d) Find the exact values of $x$ for which $\left|\frac{2}{x-3}\right| = 3$.

**(3)**



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**4.** The function f is defined by

$$\mathrm{f} : x \mapsto \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \quad x > 3.$$

(a) Show that $\mathrm{f}(x) = \dfrac{1}{x+1}, \quad x > 3.$ **(4)**

(b) Find the range of f. **(2)**

(c) Find $\mathrm{f}^{-1}(x)$. State the domain of this inverse function. **(3)**

The function g is defined by

$$\mathrm{g} : x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(d) Solve $\mathrm{fg}(x) = \dfrac{1}{8}$. **(3)**



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**7.** The function f is defined by

$$\mathrm{f}: x \mapsto \frac{3(x+1)}{2x^2+7x-4} - \frac{1}{x+4}, \qquad x \in \mathbb{R}, x > \frac{1}{2}$$

(a) Show that $\mathrm{f}(x) = \dfrac{1}{2x-1}$ **(4)**

(b) Find $\mathrm{f}^{-1}(x)$ **(3)**

(c) Find the domain of $\mathrm{f}^{-1}$ **(1)**

$$\mathrm{g}(x) = \ln(x+1)$$

(d) Find the solution of $\mathrm{fg}(x) = \dfrac{1}{7}$, giving your answer in terms of e. **(4)**



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**6.** The function f is defined by

$$\mathrm{f} : x \mapsto \ln(4-2x), \quad x<2 \quad \text{and} \quad x \in \mathbb{R}.$$

(a) Show that the inverse function of f is defined by

$$\mathrm{f}^{-1} : x \mapsto 2-\frac{1}{2}\mathrm{e}^{x}$$

and write down the domain of $\mathrm{f}^{-1}$.

**(4)**

(b) Write down the range of $\mathrm{f}^{-1}$.

**(1)**

(c) In the space provided on page 16, sketch the graph of $y=\mathrm{f}^{-1}(x)$. State the coordinates of the points of intersection with the $x$ and $y$ axes.

**(4)**

The graph of $y=x+2$ crosses the graph of $y=\mathrm{f}^{-1}(x)$ at $x=k$.

The iterative formula

$$x_{n+1}=-\frac{1}{2}\mathrm{e}^{x_n}, \quad x_0=-0.3$$

is used to find an approximate value for $k$.

(d) Calculate the values of $x_1$ and $x_2$, giving your answers to 4 decimal places.

**(2)**

(e) Find the value of $k$ to 3 decimal places.

**(2)**



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