C3
Chapter 1
Algebraic Fractions

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1. Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)},$$

find the values of the constants a, b, c, d and e.

(4)

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2. $$f(x) = \frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3}$$

(a) Express $f(x)$ as a single fraction in its simplest form.

(4)

(b) Hence show that $f'(x) = \frac{2}{(x-3)^2}$

(3)

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1. Express

$$\frac{x+1}{3x^2-3} - \frac{1}{3x+1}$$

as a single fraction in its simplest form.

(4)

Leave blank

2. (a) Express

$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

as a single fraction in its simplest form.

(4)

Given that

$$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \qquad x > 1,$$

(b) show that

$$f(x) = \frac{3}{2x-1}$$

(2)

(c) Hence differentiate $f(x)$ and find $f'(2)$.

(3)

Leave blank

7. $$f(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9}, \qquad x \neq \pm 3,\ x \neq -\frac{1}{2}$$

(a) Show that

$$f(x) = \frac{5}{(2x+1)(x+3)}$$

(5)

The curve C has equation $y = f(x)$. The point $P\left(-1, -\frac{5}{2}\right)$ lies on C.

(b) Find an equation of the normal to C at P.

(8)

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2. $$f(x) = \frac{2x+3}{x+2} - \frac{9+2x}{2x^2+3x-2}, \qquad x > \frac{1}{2}.$$

(a) Show that $f(x) = \frac{4x-6}{2x-1}$.

(7)

(b) Hence, or otherwise, find $f'(x)$ in its simplest form.

(3)

Leave blank

7. The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R},\ x \neq -4,\ x \neq 2$$

(a) Show that $f(x) = \dfrac{x-3}{x-2}$

(5)

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R},\ x \neq \ln 2$$

(b) Differentiate $g(x)$ to show that $g'(x) = \dfrac{e^x}{(e^x - 2)^2}$

(3)

(c) Find the exact values of x for which $g'(x) = 1$

(4)

Leave blank

8. (a) Simplify fully

$$\frac{2x^2+9x-5}{x^2+2x-15}$$

(3)

Given that

$$\ln(2x^2+9x-5)=1+\ln(x^2+2x-15)\,, \quad x\neq -5,$$

(b) find x in terms of e.

(4)

Leave blank

2. $$f(x) = 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2}, \quad x \neq -2.$$

(a) Show that $f(x) = \frac{x^2 + x + 1}{(x+2)^2}, \quad x \neq -2.$

(4)

(b) Show that $x^2 + x + 1 > 0$ for all values of x.

(3)

(c) Show that $f(x) > 0$ for all values of x, $x \neq -2$.

(1)