Integration

Leave blank

1. $$f(x) = x^3 + 3x^2 + 5.$$

Find

(a) $f''(x)$,

(3)

(b) $\int_1^2 f(x)\,dx$.

(4)

Leave blank

2. (a) In the space provided, sketch the graph of $y = 3^x$, $x \in \mathbb{R}$, showing the coordinates of the point at which the graph meets the y-axis.

(2)

(b) Complete the table, giving the values of 3^x to 3 decimal places.

x	0	0.2	0.4	0.6	0.8	1
3^x		1.246	1.552			3

(2)

(c) Use the trapezium rule, with all the values from your table, to find an approximation for the value of $\int_0^1 3^x \mathrm{d}x$.

(4)

N 2 3 5 5 8 A 0 8 2 0

Leave blank

3.

Figure 3

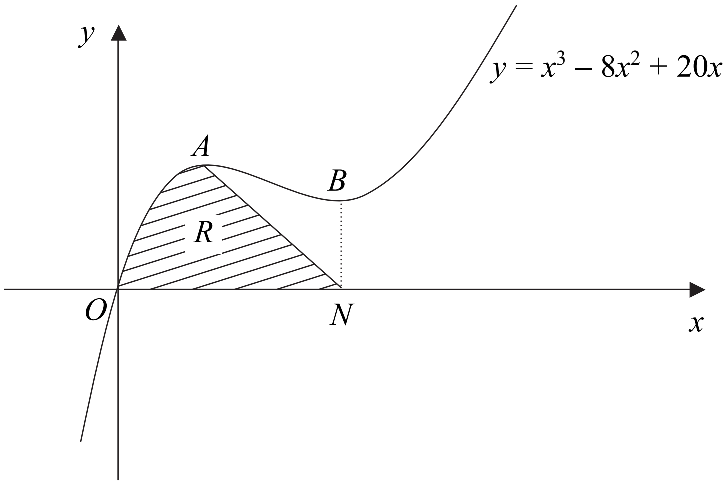


Figure 3 shows a sketch of part of the curve with equation $y = x^3 - 8x^2 + 20x$.
The curve has stationary points A and B.

(a) Use calculus to find the x-coordinates of A and B.

(4)

(b) Find the value of $\frac{d^2y}{dx^2}$ at A, and hence verify that A is a maximum.

(2)

The line through B parallel to the y-axis meets the x-axis at the point N.
The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line from A to N.

(c) Find $\int (x^3 - 8x^2 + 20x)\,dx$.

(3)

(d) Hence calculate the exact area of R.

(5)

Leave blank

4. Evaluate $\int_1^8 \frac{1}{\sqrt{x}}\,\mathrm{d}x$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

(4)

Q1

(Total 4 marks)

Leave blank

5. The curve C has equation

$$y = x\sqrt{(x^3+1)}, \qquad 0 \leqslant x \leqslant 2.$$

(a) Complete the table below, giving the values of y to 3 decimal places at $x = 1$ and $x = 1.5$.

x	0	0.5	1	1.5	2
y	0	0.530			6

(2)

(b) Use the trapezium rule, with all the y values from your table, to find an approximation for the value of $\int_0^2 x\sqrt{(x^3+1)}\,dx$, giving your answer to 3 significant figures.

(4)

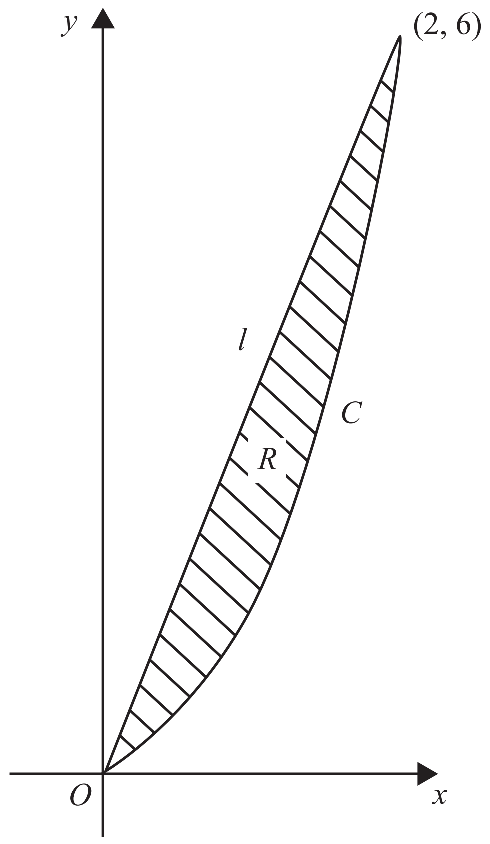


Figure 2

Figure 2 shows the curve C with equation $y = x\sqrt{(x^3+1)}$, $0 \leqslant x \leqslant 2$, and the straight line segment l, which joins the origin and the point (2, 6). The finite region R is bounded by C and l.

(c) Use your answer to part (b) to find an approximation for the area of R, giving your answer to 3 significant figures.

(3)

Leave blank

6. $$y = \sqrt{(5^x + 2)}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.5	1	1.5	2
y			2.646	3.630	

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximation for the value of $\int_0^2 \sqrt{(5^x + 2)}\,dx$.

(4)

Leave blank

7.

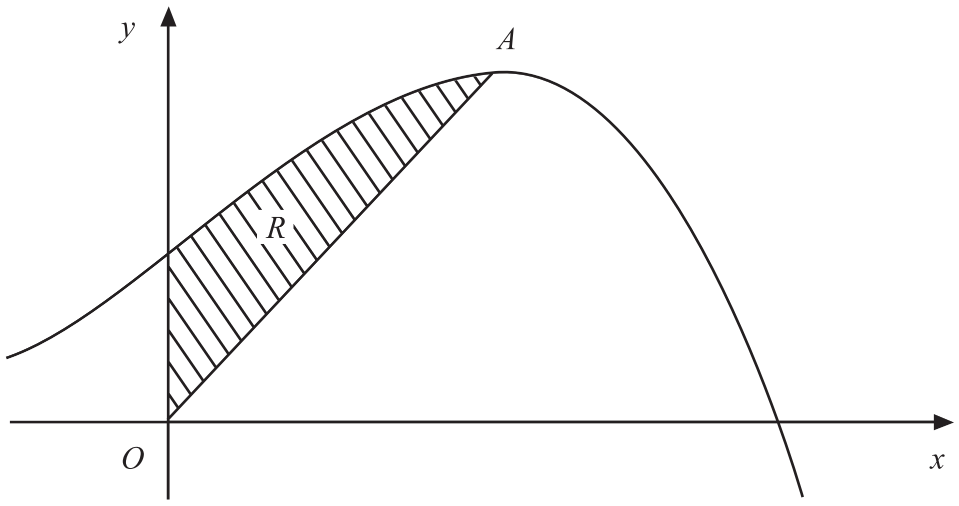


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 10 + 8x + x^2 - x^3$.

The curve has a maximum turning point A.

(a) Using calculus, show that the x-coordinate of A is 2.

(3)

The region R, shown shaded in Figure 2, is bounded by the curve, the y-axis and the line from O to A, where O is the origin.

(b) Using calculus, find the exact area of R.

(8)

Leave blank

8. Use calculus to find the value of

$$\int_1^4 \left(2x + 3\sqrt{x}\right) \mathrm{d}x\,.$$

(5)

(Total 5 marks)

Q1

H 3 4 2 6 3 A 0 3 2 4

Turn over

Leave blank

9. (a) Complete the table below, giving values of $\sqrt{(2^x + 1)}$ to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
$\sqrt{(2^x + 1)}$	1.414	1.554	1.732	1.957			3

(2)

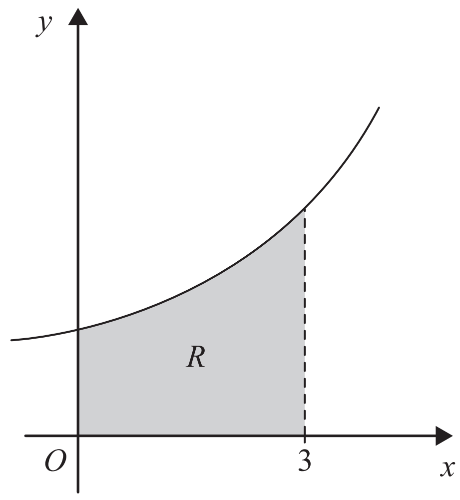


Figure 1

Figure 1 shows the region R which is bounded by the curve with equation $y = \sqrt{(2^x + 1)}$, the x-axis and the lines $x = 0$ and $x = 3$

(b) Use the trapezium rule, with all the values from your table, to find an approximation for the area of R.

(4)

(c) By reference to the curve in Figure 1 state, giving a reason, whether your approximation in part (b) is an overestimate or an underestimate for the area of R.

(2)

H 3 4 2 6 3 A 0 8 2 4

Leave blank

10.

$$y = 3^x + 2x$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	0	0.2	0.4	0.6	0.8	1
y	1	1.65				5

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_0^1 (3^x + 2x)\, dx$.

(4)

Leave blank

11.

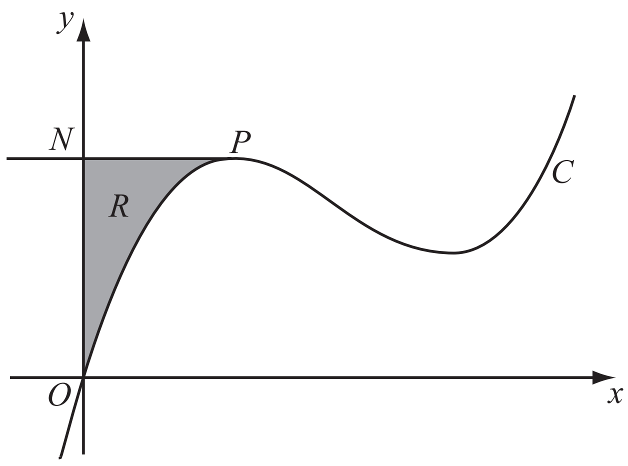


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$.

(3)

The line through P parallel to the x-axis cuts the y-axis at the point N.
The region R is bounded by C, the y-axis and PN, as shown shaded in Figure 2.

(b) Use calculus to find the exact area of R.

(6)

H 3 5 3 8 4 A 0 1 8 2 8

Leave blank

12.

Figure 1

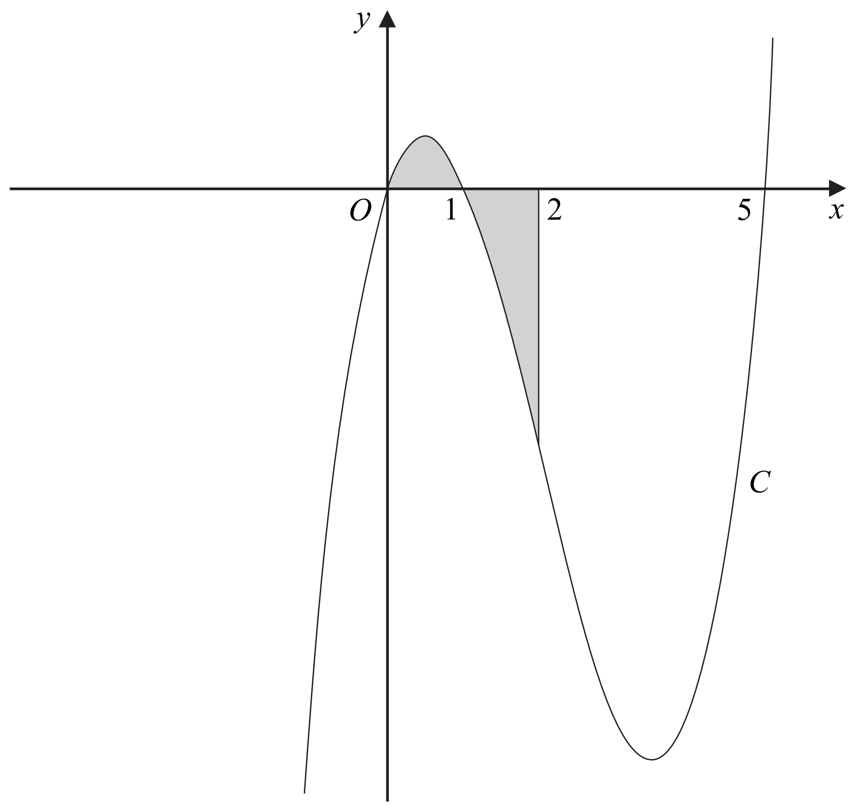


Figure 1 shows a sketch of part of the curve C with equation

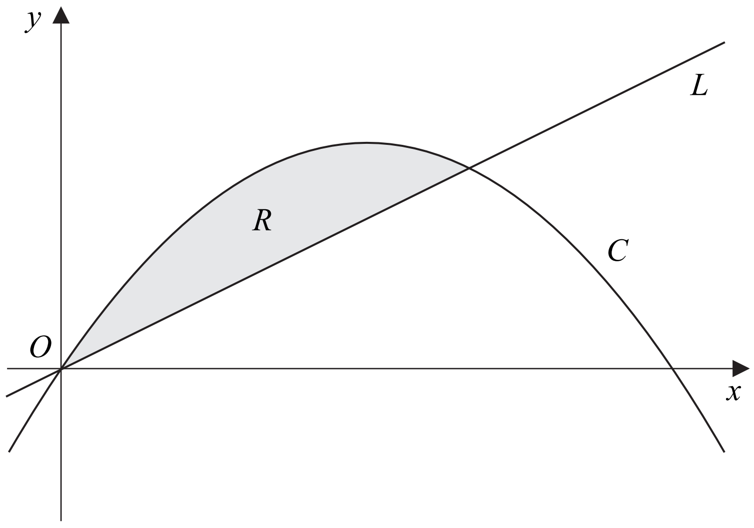
$$y = x(x - 1)(x - 5).$$

Use calculus to find the total area of the finite region, shown shaded in Figure 1, that is between $x = 0$ and $x = 2$ and is bounded by C, the x-axis and the line $x = 2$.

(9)

13.

Figure 2



In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects the x-axis at $x = 0$ and $x = 6$.

(1)

(b) Show that the line L intersects the curve C at the points (0, 0) and (4, 8).

(3)

The region R, bounded by the curve C and the line L, is shown shaded in Figure 2.

(c) Use calculus to find the area of R.

(6)

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14.

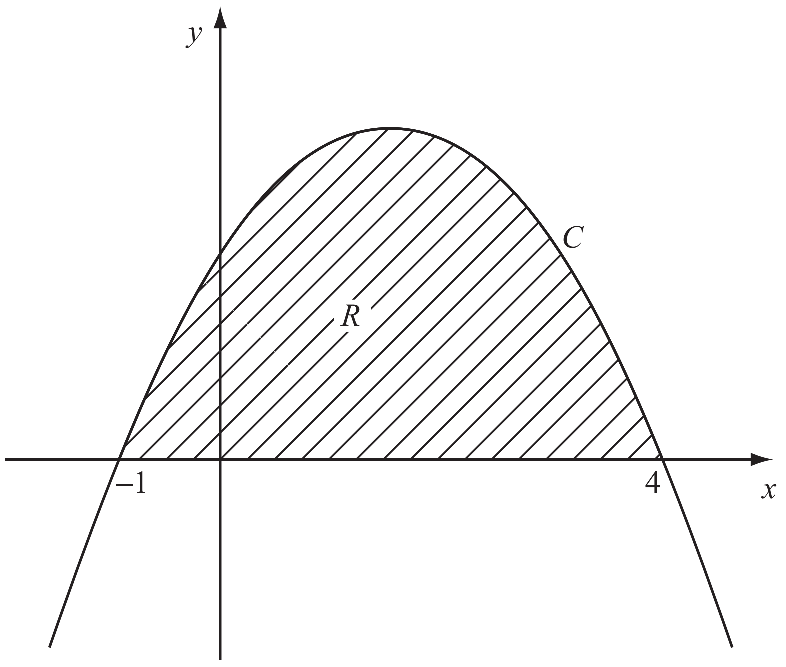

Figure 1

Figure 1 shows part of the curve C with equation $y = (1 + x)(4 - x)$.

The curve intersects the x-axis at $x = -1$ and $x = 4$. The region R, shown shaded in Figure 1, is bounded by C and the x-axis.

Use calculus to find the exact area of R.

(5)

H 3 0 9 5 7 A 0 4 2 8

Leave blank

15. $$y = \sqrt{(10x - x^2)}$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	1	1.4	1.8	2.2	2.6	3
y	3	3.47			4.39	

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximation for the value of $\int_1^3 \sqrt{(10x - x^2)}\,dx$.

(4)

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16.

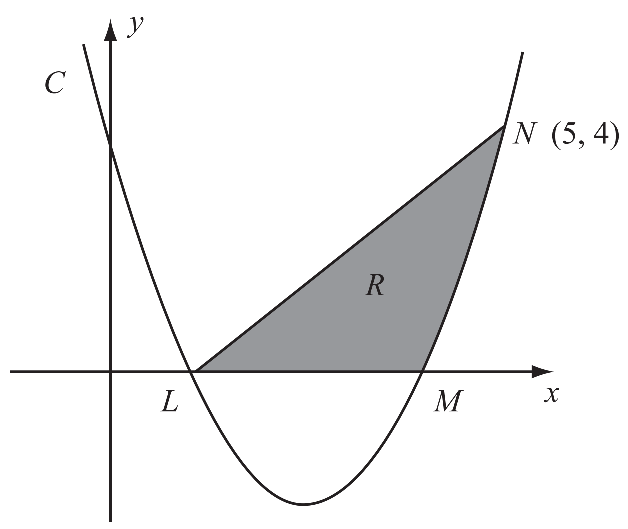

Figure 2

The curve C has equation $y = x^2 - 5x + 4$. It cuts the x-axis at the points L and M as shown in Figure 2.

(a) Find the coordinates of the point L and the point M.

(2)

(b) Show that the point N (5, 4) lies on C.

(1)

(c) Find $\int (x^2 - 5x + 4)\,dx$.

(2)

The finite region R is bounded by LN, LM and the curve C as shown in Figure 2.

(d) Use your answer to part (c) to find the exact value of the area of R.

(5)

17.

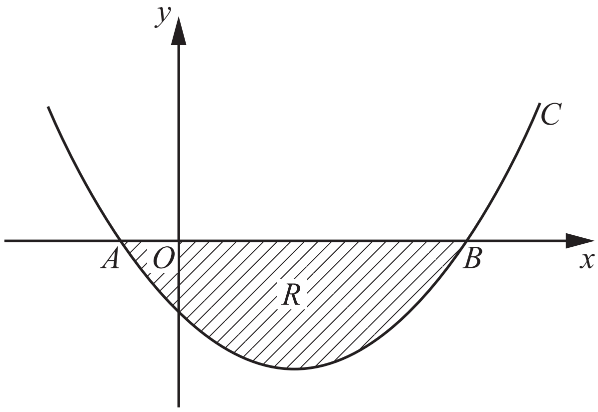

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x+1)(x-5)$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B.

(1)

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R.

(6)

Leave blank

18. $$y = \frac{5}{3x^2 - 2}$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	2	2.25	2.5	2.75	3
y	0.5	0.38			0.2

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_2^3 \frac{5}{3x^2 - 2}\,\mathrm{d}x$.

(4)

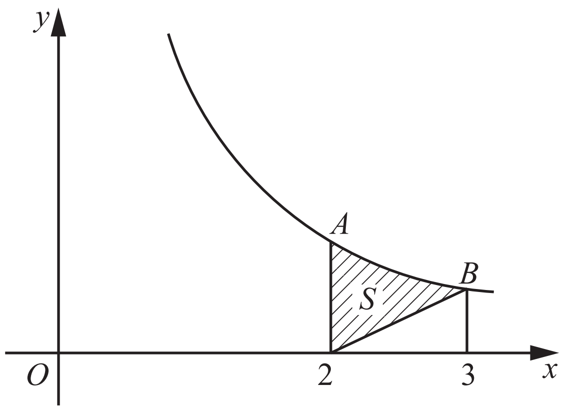

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \frac{5}{3x^2 - 2}$, $x > 1$.

At the points A and B on the curve, $x = 2$ and $x = 3$ respectively.

The region S is bounded by the curve, the straight line through B and (2, 0), and the line through A parallel to the y-axis. The region S is shown shaded in Figure 2.

(c) Use your answer to part (b) to find an approximate value for the area of S.

(3)

Leave blank

19. Use calculus to find the exact value of $\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2}\right) \mathrm{d}x$.

(5)

Q2

(Total 5 marks)

N 2 3 5 5 8 A 0 4 2 0