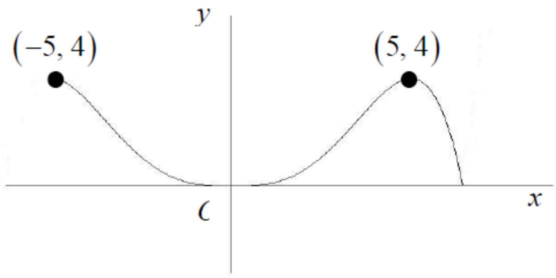
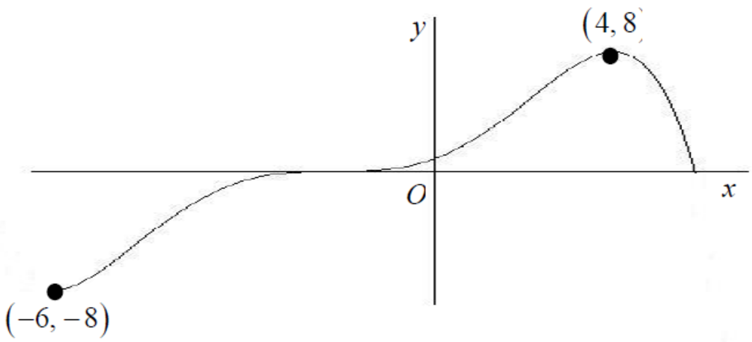
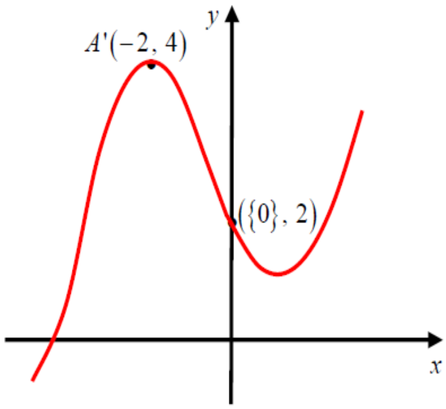
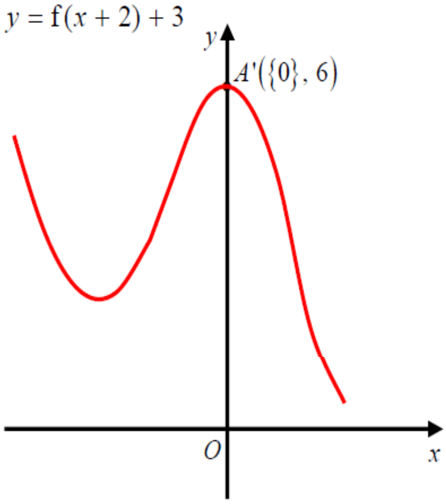
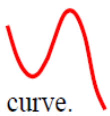
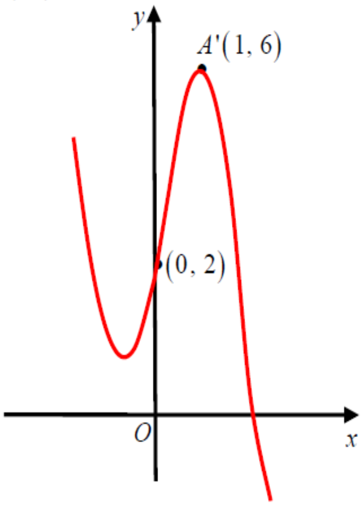
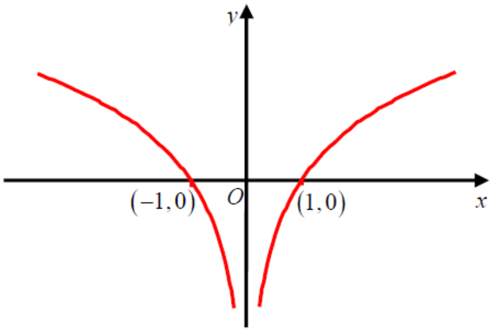
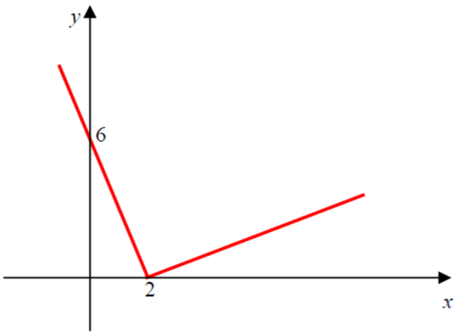
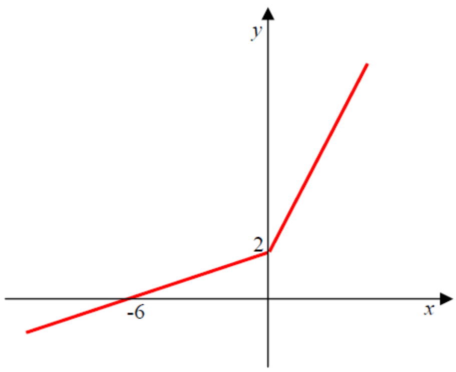
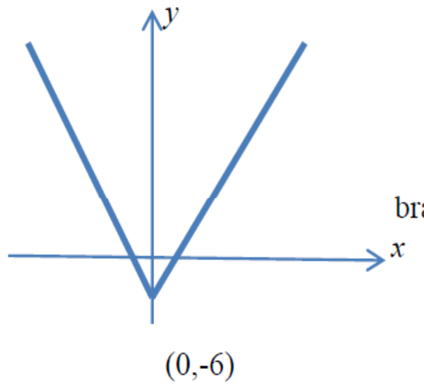
C3
Chapter 5
Transforming graphs of functions

Question Number	Scheme		Marks	
4.	(a)			
				
		Shape	B1	
		$(5, 4)$	B1	
		$(-5, 4)$	B1	(3)
	(b) For the purpose of marking this paper, the graph is identical to (a)			
		Shape	B1	
		$(5, 4)$	B1	
		$(-5, 4)$	B1	(3)
	(c)			
				
		General shape – unchanged	B1	
		Translation to left	B1	
		$(4, 8)$	B1	
		$(-6, -8)$	B1	(4)
	In all parts of this question ignore any drawing outside the domains shown in the diagrams above.			**[10]**

Question Number	Scheme	Marks
Q6 (i)	$y = \mathrm{f}(-x) + 1$ 	
	Shape of  and must have a maximum in quadrant 2 and a minimum in quadrant 1 or on the positive y-axis.	B1
	Either $(\{0\}, 2)$ or $A'(-2, 4)$	B1
	Both $(\{0\}, 2)$ and $A'(-2, 4)$	B1
		(3)
(ii)	$y = \mathrm{f}(x+2) + 3$ 	
	Any translation of the original curve. 	B1
	The ***translated maximum*** has either x-coordinate of 0 (can be implied) or y-coordinate of 6.	B1
	The translated curve has maximum $(\{0\}, 6)$ and is in the correct position on the Cartesian axes.	B1
		(3)
(iii)	$y = 2\mathrm{f}(2x)$ 	
	Shape of  with a minimum in quadrant 2 and a maximum in quadrant 1.	B1
	Either $(\{0\}, 2)$ or $A'(1, 6)$	B1
	Both $(\{0\}, 2)$ and $A'(1, 6)$	B1
		(3)
		[9]

Question Number	Scheme		Marks
Q5	$y = \ln\|x\|$		
		Right-hand branch in quadrants 4 and 1. Correct shape.	B1
		Left-hand branch in quadrants 2 and 3. Correct shape.	B1
		Completely correct sketch and both $(-1, \{0\})$ and $(1, \{0\})$	B1
			(3)
			[3]

Question Number	Scheme		Marks
6.			
(a)	$y = \dfrac{3-2x}{x-5} \Rightarrow y(x-5) = 3-2x$	Attempt to make x (or swapped y) the subject	M1
	$xy - 5y = 3 - 2x$		
	$\Rightarrow xy + 2x = 3 + 5y \Rightarrow x(y+2) = 3+5y$	Collect x terms together and factorise.	M1
	$\Rightarrow x = \dfrac{3+5y}{y+2} \quad \therefore \mathrm{f}^{-1}(x) = \underline{\dfrac{3+5x}{x+2}}$	$\underline{\dfrac{3+5x}{x+2}}$	A1 oe
			(3)
(b)	Range of g is $-9 \le \mathrm{g}(x) \le 4$ or $-9 \le y \le 4$	<u>Correct Range</u>	B1 (1)
(c)		Deduces that $\mathrm{g}(2)$ is 0. Seen or implied.	M1
	$\mathrm{g\,g}(2) = \mathrm{g}(0) = -6$, from sketch.	-6	A1 (2)
(d)	$\mathrm{fg}(8) = \mathrm{f}(4)$	Correct order g followed by f	M1
	$= \dfrac{3-4(2)}{4-5} = \dfrac{-5}{-1} = \underline{5}$	5	A1
			(2)
(e)(i)		Correct shape 	B1
		$(2, \{0\}), (\{0\}, 6)$	B1
(e)(ii)		Correct shape 	B1
		Graph goes through $(\{0\}, 2)$ and $(-6, \{0\})$ which are marked.	B1
			(4)
(f)	Domain of g^{-1} is $-9 \le x \le 4$	Either correct answer or a follow through from part (b) answer	B1 $\sqrt{}$
			(1) **[13]**

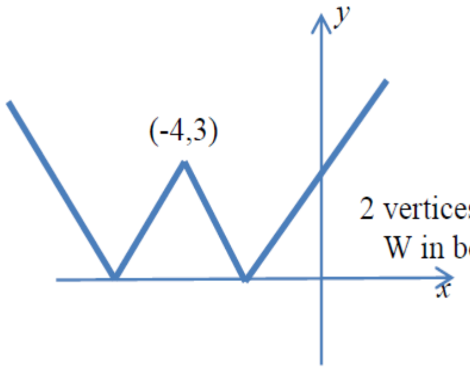
<table>
<tr><th>Question Number</th><th>Scheme</th><th></th><th>Marks</th></tr>
<tr><td>3 (a)</td><td>
</td><td>V shape

vertex on y axis &both branches of graph cross x axis

‘y’ co-ordinate of R is -6</td><td>B1

B1

B1

(3)</td></tr>
<tr><td>(b)</td><td>
</td><td>W shape

2 vertices on the negative x axis. W in both quad 1 & quad 2.

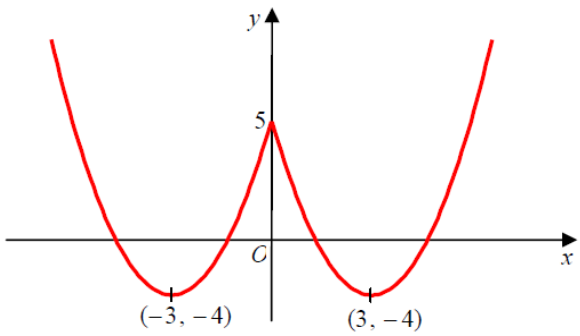
R’=(-4,3)</td><td>B1

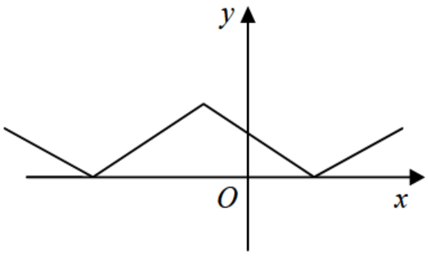
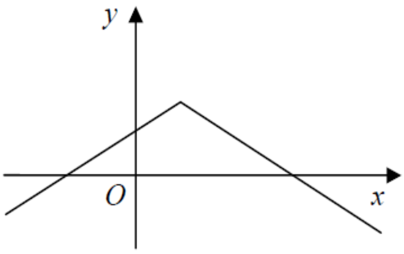
B1dep

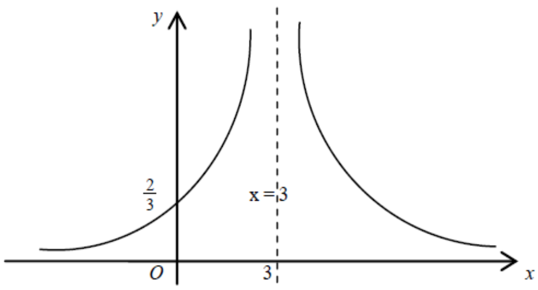
B1

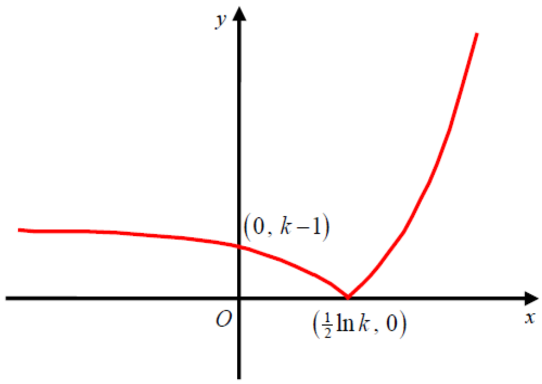
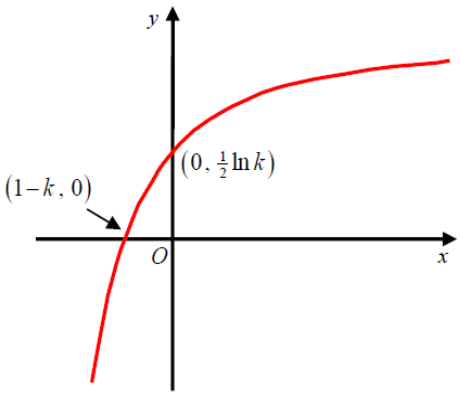
(3)

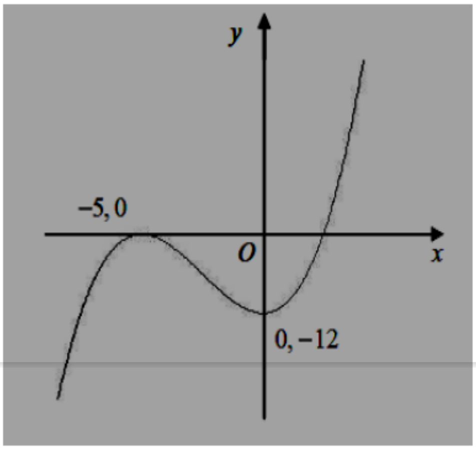
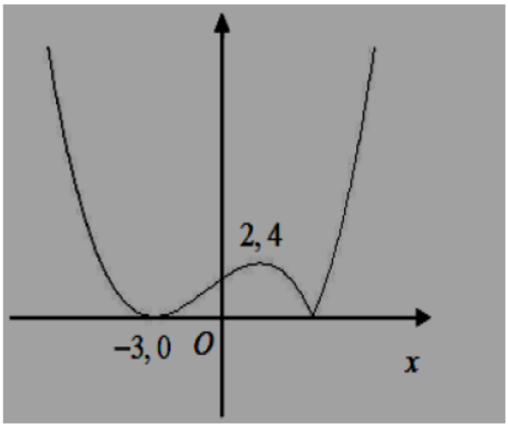
6 Marks</td></tr>
</table>

Question Number	Scheme	Marks
6. (a) (i)	$(3, 4)$	B1 B1
(ii)	$(6, -8)$	B1 B1 (4)
(b)		B1 B1 B1 (3)
(c)	$\mathrm{f}(x) = (x-3)^2 - 4$ or $\mathrm{f}(x) = x^2 - 6x + 5$	M1A1 (2)
(d)	Either: The function f is a many-one {mapping}. Or: The function f is not a one-one {mapping}.	B1 (1) **[10]**
	(b) B1: Correct shape for $x \geqslant 0$, with the curve meeting the positive y-axis and the turning point is found below the x-axis. (providing candidate does not copy the whole of the original curve and adds nothing else to their sketch.). B1: Curve is symmetrical about the y-axis or correct shape of curve for $x < 0$. **Note:** The first two B1B1 can only be awarded if the curve has the correct shape, with a cusp on the positive *y*-axis and with both turning points located in the correct quadrants. Otherwise award B1B0. B1: Correct turning points of $(-3, -4)$ and $(3, -4)$. Also, $(\{0\}, 5)$ is marked where the graph cuts through the y-axis. Allow $(5, 0)$ rather than $(0, 5)$ if marked in the "correct" place on the y-axis. (c) M1: Either states $\mathrm{f}(x)$ in the form $(x \pm \alpha)^2 \pm \beta$; $\alpha, \beta \neq 0$ Or uses a complete method on $\mathrm{f}(x) = x^2 + ax + b$, with $\mathrm{f}(0) = 5$ and $\mathrm{f}(3) = -4$ to find both *a* and *b*. A1: Either $(x-3)^2 - 4$ or $x^2 - 6x + 5$ (d) B1: Or: The inverse is a one-many {mapping and not a function}. Or: Because $\mathrm{f}(0) = 5$ and also $\mathrm{f}(6) = 5$. Or: One y-coordinate has 2 corresponding x-coordinates {and therefore cannot have an inverse}.	

Question Number	Scheme	Marks
3.	(a) 	
	 shape	B1
	Vertices correctly placed	B1 (2)
	(b) 	
	 shape	B1
	Vertex and intersections with axes correctly placed	B1 (2)
	(c) $P:(-1,2)$	B1
	$Q:(0,1)$	B1
	$R:(1,0)$	B1 (3)
	(d) $x>-1;\quad 2-x-1=\frac{1}{2}x$	M1 A1
	Leading to $x=\frac{2}{3}$	A1
	$x<-1;\quad 2+x+1=\frac{1}{2}x$	M1
	Leading to $x=-6$	A1 (5)
		[12]

5. (*a*)	Finding g(4) = k and f(k) = …. or $\text{fg}(x) = \ln\left(\dfrac{4}{x-3} - 1\right)$		M1
	[f(2) = ln(2x2 – 1) fg(4) = ln(4 – 1)] = ln 3		A1 (2)
(*b*)	$y = \ln(2x-1) \;\Rightarrow\; e^y = 2x-1$ or $e^x = 2y - 1$		M1, A1
	$f^{-1}(x) = \frac{1}{2}(e^x + 1)$ Allow $y = \frac{1}{2}(e^x + 1)$		A1
	Domain $x \in \Re$ [Allow $\Re$, all reals, $(-\infty, \infty)$] independent		B1 (4)
(*c*)		Shape, and x-axis should appear to be asymptote	B1
		Equation $x = 3$ needed, may see in diagram (ignore others)	B1 ind.
		Intercept $(0, \frac{2}{3})$ no other; accept y = ⅔ (0.67) or on graph	B1 ind (3)
(*d*)	$\dfrac{2}{x-3} = 3 \quad \Rightarrow x = 3\frac{2}{3}$ or exact equiv.		B1
	$\dfrac{2}{x-3} = -3, \quad \Rightarrow x = 2\frac{1}{3}$ or exact equiv.		M1, A1 (3)
	Note: $2 = 3(x + 3)$ or $2 = 3(-x - 3)$ o.e. is M0A0		
Alt:	Squaring to quadratic $(9x^2 - 54x + 77 = 0)$ **and** solving M1; B1A1		**(12 marks)**

Question Number	Scheme		Marks
Q5 (a)		Curve retains shape when $x > \tfrac{1}{2}\ln k$	B1
		Curve reflects through the x-axis when $x < \tfrac{1}{2}\ln k$	B1
		$(0,\ k-1)$ and $\left(\tfrac{1}{2}\ln k,\ 0\right)$ marked in the correct positions.	B1
			(3)
(b)		Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote)	B1
		$(1-k,\ 0)$ and $\left(0,\ \tfrac{1}{2}\ln k\right)$	B1
			(2)
(c)	Range of f: $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k,\ \infty)}$	Either $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k,\ \infty)}$ or $\underline{f > -k}$ or $\underline{\text{Range} > -k}$.	B1
			(1)
(d)	$y = e^{2x} - k \Rightarrow y + k = e^{2x}$	Attempt to make x (or swapped y) the subject	M1
	$\Rightarrow \ln(y+k) = 2x$ $\Rightarrow \tfrac{1}{2}\ln(y+k) = x$	Makes e^{2x} the subject and takes ln of both sides	M1
	Hence $f^{-1}(x) = \underline{\tfrac{1}{2}\ln(x+k)}$	$\underline{\tfrac{1}{2}\ln(x+k)}$ or $\underline{\ln\sqrt{(x+k)}}$	$\underline{\text{A1}}$ cao
			(3)
(e)	$f^{-1}(x)$: Domain: $\underline{x > -k}$ or $\underline{(-k,\ \infty)}$	Either $\underline{x > -k}$ or $\underline{(-k,\ \infty)}$ or Domain $> -k$ or x "ft one sided inequality" their part (c) RANGE answer	B1 $\sqrt{}$
			(1)
			[10]

Question No	Scheme		Marks
2	(a)		
		Shape	B1
		x coordinates correct	B1
		y coordinates correct	B1
			(3)
	(b)		
		Shape	B1
		Max at (2,4)	B1
		Min at (-3,0)	B1
			(3)
			6 marks