C3
Chapter 4
Numerical Methods

Question Number	Scheme		Marks	
3.	(a) $f(2) = 0.38 \ \ldots$			
	$f(3) = -0.39 \ \ldots$		M1	
	Change of sign (and continuity) $\Rightarrow$ root in $(2, 3)$ $\ast$	cso	A1	(2)
	(b) $x_1 = \ln 4.5 + 1 \approx 2.50408$		M1	
	$x_2 \approx 2.50498$		A1	
	$x_3 \approx 2.50518$		A1	(3)
	(c) Selecting $[2.5045, 2.5055]$, or appropriate tighter range, and evaluating at both ends.		M1	
	$f(2.5045) \approx 6 \times 10^{-4}$			
	$f(2.5055) \approx -2 \times 10^{-4}$			
	Change of sign (and continuity) $\Rightarrow$ root $\in (2.5045, 2.5055)$			
	$\Rightarrow$ root $= 2.505$ to 3 dp $\ast$	cso	A1	(2)
				[7]
	Note: The root, correct to 5 dp, is 2.50524			

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Question Number	Scheme		Marks
Q2	$\mathrm{f}(x) = x^3 + 2x^2 - 3x - 11$		
(a)	$\mathrm{f}(x) = 0 \Rightarrow x^3 + 2x^2 - 3x - 11 = 0$ $\Rightarrow x^2(x+2) - 3x - 11 = 0$	Sets $\mathrm{f}(x) = 0$ (can be implied) and takes out a factor of x^2 from $x^3 + 2x^2$, or x from $x^3 + 2x$ (slip).	M1
	$\Rightarrow x^2(x+2) = 3x + 11$ $\Rightarrow x^2 = \dfrac{3x+11}{x+2}$ $\Rightarrow x = \sqrt{\left(\dfrac{3x+11}{x+2}\right)}$	then rearranges to give the quoted result on the question paper.	A1 AG
			(2)
(b)	Iterative formula: $x_{n+1} = \sqrt{\left(\dfrac{3x_n+11}{x_n+2}\right)}$, $x_1 = 0$		
	$x_2 = \sqrt{\left(\dfrac{3(0)+11}{(0)+2}\right)}$	An attempt to substitute $x_1 = 0$ into the iterative formula. Can be implied by $x_2 = \sqrt{5.5}$ ***or 2.35*** or awrt 2.345	M1
	$x_2 = 2.34520788...$ $x_3 = 2.037324945...$	Both $x_2 =$ awrt 2.345 and $x_3 =$ awrt 2.037	A1
	$x_4 = 2.058748112...$	$x_4 =$ awrt 2.059	A1
			(3)
(c)	Let $\mathrm{f}(x) = x^3 + 2x^2 - 3x - 11 = 0$		
	$\mathrm{f}(2.0565) = -0.013781637...$ $\mathrm{f}(2.0575) = 0.0041401094...$	Choose suitable interval for x, e.g. [2.0565, 2.0575] or tighter	M1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α is	any one value awrt 1 sf	dM1
	such that $\alpha \in (2.0565, 2.0575) \Rightarrow \alpha = 2.057$ (3 dp)	both values correct awrt 1sf, sign change and conclusion	A1
			(3)
		As a minimum, both values must be correct to 1 sf, candidate states "change of sign, hence root".	
			[8]

2 (a)	f(0.75)= -0.18… f(0.85) = 0.17…..	M1	
	Change of sign, hence root between x=0.75 and x=0.85	A1	(2)
(b)	Sub x_0=0.8 into $x_{n+1} = [\arcsin(1 - 0.5x_n)]^{\frac{1}{2}}$ to obtain x_1	M1	
	Awrt x_1=0.80219 and x_2=0.80133	A1	
	Awrt x_3 = 0.80167	A1	(3)
(c)	f(0.801565) = -2.7…..$\times 10^{-5}$ f(0.801575) = +8.6….$\times 10^{-6}$	M1A1	
	Change of sign and conclusion	A1	(3)

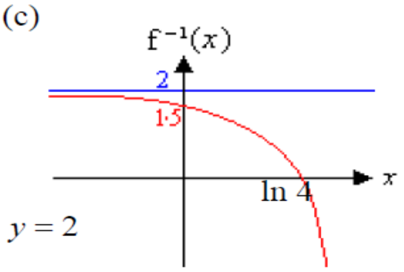
Question Number	Scheme	Marks
3. (a)	$\text{f}(1.2) = 0.49166551...,\quad \text{f}(1.3) = -0.048719817...$ Sign change (and as $\text{f}(x)$ is continuous) therefore a root α is such that $\alpha \in [1.2, 1.3]$	M1A1 (2)
(b)	$4\text{cosec}x - 4x + 1 = 0 \Rightarrow 4x = 4\text{cosec}x + 1$ $\Rightarrow x = \text{cosec}x + \frac{1}{4} \Rightarrow x = \frac{1}{\sin x} + \frac{1}{4}$	M1 A1 * (2)
(c)	$x_1 = \frac{1}{\sin(1.25)} + \frac{1}{4}$ $x_1 = 1.303757858...,\quad x_2 = 1.286745793...$ $x_3 = 1.291744613...$	M1 A1 A1 (3)
(d)	$\text{f}(1.2905) = 0.00044566695...,\quad \text{f}(1.2915) = -0.00475017278...$ Sign change (and as $\text{f}(x)$ is continuous) therefore a root α is such that $\alpha \in (1.2905, 1.2915) \Rightarrow \alpha = 1.291$ (3 dp)	M1 A1 (2) **[9]**
	(a) M1: Attempts to evaluate both $\text{f}(1.2)$ and $\text{f}(1.3)$ and evaluates at least one of them correctly to awrt (or truncated) 1 sf. A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion. (b) M1: Attempt to make $4x$ or x the subject of the equation. A1: Candidate must then rearrange the equation to give the required result. It must be clear that candidate has made their initial $\text{f}(x) = 0$. (c) M1: An attempt to substitute $x_0 = 1.25$ into the iterative formula Eg $= \frac{1}{\sin(1.25)} + \frac{1}{4}$. Can be implied by x_1 = awrt 1.3 or x_1 = awrt 46°. A1: Both x_1 = awrt 1.3038 and x_2 = awrt 1.2867 A1: x_3 = awrt 1.2917 (d) M1: Choose suitable interval for x, e.g. [1.2905, 1.2915] or tighter and at least one attempt to evaluate $\text{f}(x)$. A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion.	

4.	(*a*)	$x^2(3-x)-1=0$ o.e. (e.g. $x^2(-x+3)=1$) $x=\sqrt{\dfrac{1}{3-x}}$ (✱) Note(✱), answer is given: need to see appropriate working and A1 is cso [Reverse process: Squaring and non-fractional equation M1, form f(x) A1]	M1 A1 (cso) (2)
	(*b*)	$x_2=0.6455$, $x_3=0.6517$, $x_4=0.6526$ 1st B1 is for one correct, 2nd B1 for other two correct If all three are to greater accuracy, award B0 B1	B1; B1 (2)
	(*c*)	Choose values in interval (0.6525, 0.6535) or tighter and evaluate both f(0.6525) = –0.0005 (372… f(0.6535) = 0.002 (101… At least one correct "up to bracket", i.e. -0.0005 or 0.002 **Change of sign,** ∴ $x=0.653$ is a root (correct) to 3 d.p. Requires both correct "up to bracket" and conclusion as above	M1 A1 A1 (3) **(7 marks)**
	Alt (i)	Continued iterations at least as far as x_6 M1 $x_5=0.65268$, $x_6=0.6527$, $x_7=$ … two correct to at least 4 s.f. A1 Conclusion : Two values correct to 4 d.p., so 0.653 is root to 3 d.p. A1	
	Alt (ii)	If use g(0.6525) = 0.6527..>0.6525 and g(0.6535) = 0.6528..<0.6535 M1A1 Conclusion : Both results correct, so 0.653 is root to 3 d.p. A1	

Question Number	Scheme	Marks
7.	(a) $f(1.4)=-0.568\ \ldots<0$ $f(1.45)=0.245\ \ldots>0$ Change of sign (and continuity) $\Rightarrow \alpha\in(1.4, 1.45)$	 M1 A1 (2)
	(b) $3x^3=2x+6$ $x^3=\dfrac{2x}{3}+2$ $x^2=\dfrac{2}{3}+\dfrac{2}{x}$ $x=\sqrt{\left(\dfrac{2}{x}+\dfrac{2}{3}\right)}$ ∗ cso	 M1 A1 A1 (3)
	(c) $x_1=1.4371$ $x_2=1.4347$ $x_3=1.4355$	B1 B1 B1 (3)
	(d) Choosing the interval $(1.4345, 1.4355)$ or appropriate tighter interval. $f(1.4345)=-0.01$ ... $f(1.4355)=0.003$ … Change of sign (and continuity) $\Rightarrow \alpha\in(1.4345, 1.4355)$ $\Rightarrow \alpha=1.435$, correct to 3 decimal places ∗ cso *Note*: $\alpha=1.435\,304\,553$ …	M1 M1 A1 (3) **[11]**

Question Number	Scheme		Marks
Q1 (a)	Iterative formula: $x_{n+1} = \frac{2}{(x_n)^2} + 2$, $x_0 = 2.5$		
	$x_1 = \frac{2}{(2.5)^2} + 2$	An attempt to substitute $x_0 = 2.5$ into the iterative formula. Can be implied by $x_1 = 2.32$ or 2.320	M1
	$x_1 = 2.32$ $x_2 = 2.371581451...$	Both $x_1 = 2.32(0)$ and $x_2 =$ awrt 2.372	A1
	$x_3 = 2.355593575...$ $x_4 = 2.360436923...$	Both $x_3 =$ awrt 2.356 and $x_4 =$ awrt 2.360 or 2.36	A1 cso
			(3)
(b)	Let $\mathrm{f}(x) = -x^3 + 2x^2 + 2 = 0$		
	$\mathrm{f}(2.3585) = 0.00583577...$ $\mathrm{f}(2.3595) = -0.00142286...$	Choose suitable interval for x, e.g. [2.3585, 2.3595] or tighter	M1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root	any one value awrt 1 sf or truncated 1 sf	dM1
	α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)	both values correct, sign change and conclusion	A1
		At a minimum, both values must be correct to 1sf or truncated 1sf, candidate states "change of sign, hence root".	(3)
			[6]

Question No	Scheme	Marks
6	(a) f(0.8) = 0.082, f(0.9)= -0.089	M1
	Change of sign $\Rightarrow$ root (0.8,0.9)	A1
		(2)
	(b)	
	$f'(x) = 2x - 3 - \sin\left(\frac{1}{2}x\right)$	M1 A1
	Sets $f'(x) = 0 \Rightarrow \quad x = \frac{3+\sin\left(\frac{1}{2}x\right)}{2}$	M1A1*
		(4)
	(c) Sub x_0=2 into $x_{n+1} = \frac{3+\sin\left(\frac{1}{2}x_n\right)}{2}$	M1
	x_1=awrt 1.921, x_2=awrt 1.91(0) and x_3=awrt 1.908	A1,A1
		(3)
	(d) [1.90775,1.90785]	M1
	f'(1.90775)=-0.00016.. AND f'(1.90785)= 0.0000076..	M1
	Change of sign $\Rightarrow$ x=1.9078	A1
		(3)
		(12 marks)

Question Number	Scheme		Marks	
6.	(a) $y = \ln(4-2x)$			
	$e^y = 4-2x$ leading to $x = 2-\frac{1}{2}e^y$	Changing subject and removing ln	M1 A1	
	$y = 2-\frac{1}{2}e^x \quad \Rightarrow \quad f^{-1} \mapsto 2-\frac{1}{2}e^x$ *	cso	A1	
	Domain of f^{-1} is □		B1	**(4)**
	(b) Range of f^{-1} is $f^{-1}(x) < 2$ (and $f^{-1}(x) \in$ □)		B1	**(1)**
	(c) 	Shape 1.5 ln 4	B1 B1 B1 B1	 **(4)**
	(d) $x_1 \approx -0.3704, \quad x_2 \approx -0.3452$ If more than 4 dp given in this part a maximum on one mark is lost. Penalise on the first occasion.	cao	B1, B1	**(2)**
	(e) $x_3 = -0.354\,030\,19 \ldots$ $x_4 = -0.350\,926\,88 \ldots$ $x_5 = -0.352\,017\,61 \ldots$ $x_6 = -0.351\,633\,86 \ldots$	Calculating to at least x_6 to at least four dp	M1	
	$k \approx -0.352$	cao	A1	**(2)** **[13]**
	Alternative to (e) $k \approx -0.352$	Found in any way		
	Let $g(x) = x + \frac{1}{2}e^x$			
	$g(-0.3515) \approx +0.0003, \quad g(-0.3525) \approx -0.001$		M1	
	Change of sign (and continuity) $\Rightarrow k \in (-0.3525, -0.3515)$			
	$\Rightarrow k = -0.352$ (to 3 dp)		A1	**(2)**