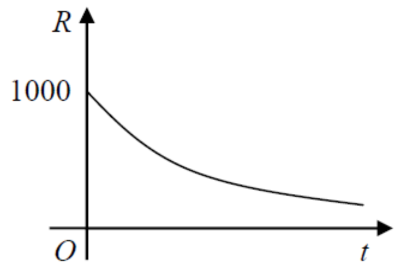
C3
Chapter 3
Exponential and log functions

Question Number	Scheme		Marks	
5.	(a) 1000		B1	(1)
	(b) $1000\,e^{-5730c} = 500$		M1	
	$e^{-5730c} = \frac{1}{2}$		A1	
	$-5730c = \ln\frac{1}{2}$		M1	
	$c = 0.000121$	cao	A1	(4)
	(c) $R = 1000\,e^{-22920c} = 62.5$	Accept 62-63	M1 A1	(2)
	(d) 	Shape 1000	B1 B1	 (2) **[9]**

Question Number	Scheme		Marks
Q9 (i)(a)	$\ln(3x-7)=5$		
	$e^{\ln(3x-7)}=e^5$	Takes e of both sides of the equation. This can be implied by $3x-7=e^5$.	M1
	$3x-7=e^5 \Rightarrow x=\dfrac{e^5+7}{3}$ $\{=51.804...\}$	Then rearranges to make x the subject.	dM1
		Exact answer of $\dfrac{e^5+7}{3}$.	A1
			(3)
(b)	$3^x e^{7x+2}=15$		
	$\ln\left(3^x e^{7x+2}\right)=\ln 15$	Takes ln (or logs) of both sides of the equation.	M1
	$\ln 3^x+\ln e^{7x+2}=\ln 15$	Applies the addition law of logarithms.	M1
	$x\ln 3+7x+2=\ln 15$	$x\ln 3+7x+2=\ln 15$	A1 oe
	$x(\ln 3+7)=-2+\ln 15$	Factorising out at least two x terms on one side and collecting number terms on the other side.	ddM1
	$x=\dfrac{-2+\ln 15}{7+\ln 3}$ $\{=0.0874...\}$	***Exact answer*** of $\dfrac{-2+\ln 15}{7+\ln 3}$	A1 oe
			(5)
(ii) (a)	$f(x)=e^{2x}+3,\ x\in\mathbb{R}$		
	$y=e^{2x}+3 \Rightarrow y-3=e^{2x}$	Attempt to make x (or swapped y) the subject	M1
	$\Rightarrow \ln(y-3)=2x$ $\Rightarrow \frac{1}{2}\ln(y-3)=x$	Makes e^{2x} the subject and takes ln of both sides	M1
	Hence $f^{-1}(x)=\underline{\frac{1}{2}\ln(x-3)}$	$\underline{\frac{1}{2}\ln(x-3)}$ or $\underline{\ln\sqrt{(x-3)}}$ or $\underline{f^{-1}(y)=\frac{1}{2}\ln(y-3)}$ (see appendix)	A1 cao
	$f^{-1}(x)$: Domain: $\underline{x>3}$ or $\underline{(3,\infty)}$	Either $\underline{x>3}$ or $\underline{(3,\infty)}$ or $\underline{\text{Domain}>3}$.	B1
			(4)
(b)	$g(x)=\ln(x-1),\ x\in\mathbb{R},\ x>1$		
	$fg(x)=e^{2\ln(x-1)}+3$ $\{=(x-1)^2+3\}$	An attempt to put function g into function f.	M1
		$e^{2\ln(x-1)}+3$ or $(x-1)^2+3$ or x^2-2x+4.	A1 isw
	$fg(x)$: Range: $\underline{y>3}$ or $\underline{(3,\infty)}$	Either $\underline{y>3}$ or $\underline{(3,\infty)}$ or $\underline{\text{Range}>3}$ or $\underline{fg(x)>3}$.	B1
			(3)
			[15]

Question Number	Scheme	Marks
1.	(a) $e^{2x+1} = 2$	
	$2x+1 = \ln 2$	M1
	$x = \frac{1}{2}(\ln 2 - 1)$	A1 (2)
	(b) $\frac{dy}{dx} = 8e^{2x+1}$	B1
	$x = \frac{1}{2}(\ln 2 - 1) \Rightarrow \frac{dy}{dx} = 16$	B1
	$y - 8 = 16\left(x - \frac{1}{2}(\ln 2 - 1)\right)$	M1
	$y = 16x + 16 - 8\ln 2$	A1 (4)
		[6]

Question No	Scheme	Marks
3	(a) 20 (mm^2)	**B1**
		(1)
	(b) $'40' = 20\ e^{1.5t} \rightarrow e^{1.5t} = c$	**M1**
	$e^{1.5t} = \frac{40}{20} = (2)$	**A1**
	Correct order $1.5t = ln'2' \rightarrow t = \frac{lnc}{1.5}$	**M1**
	$t = \frac{ln2}{1.5} = (\ awrt\ 0.46)$	**A1**
	12.28 or 28 (minutes}	**A1**
		(5)
		(6 marks)

Question Number	Scheme	Marks
1. (*a*)	$\ln 3x = \ln 6$ or $\ln x = \ln\left(\frac{6}{3}\right)$ or $\ln\left(\frac{3x}{6}\right) = 0$	M1
	$x = 2$ (only this answer)	A1 (cso) (2)
(*b*)	$(e^x)^2 - 4e^x + 3 = 0$ (any 3 term form)	M1
	$(e^x - 3)(e^x - 1) = 0$	
	$e^x = 3$ or $e^x = 1$ Solving quadratic	M1 dep
	$x = \ln 3$, $x = 0$ (or ln 1)	M1 A1 (4)
		(6 marks)

Question Number	Scheme	Marks
8. (*a*)	$D = 10, t = 5,$ $x = 10e^{-\frac{1}{8}\times 5}$	M1
	$= 5.353$ awrt	A1 (2)
(*b*)	$D = 10 + 10e^{-\frac{5}{8}}, t = 1,$ $x = 15.3526\ldots \times e^{-\frac{1}{8}}$	M1
	$x = 13.549$ (✱)	A1 cso (2)
Alt.(*b*)	$x = 10e^{-\frac{1}{8}\times 6} + 10e^{-\frac{1}{8}\times 1}$ M1 $x = 13.549$ (✱) A1 cso	
(*c*)	$15.3526...e^{-\frac{1}{8}T} = 3$	M1
	$e^{-\frac{1}{8}T} = \frac{3}{15.3526...} = 0.1954...$	
	$-\frac{1}{8}T = \ln 0.1954...$	M1
	$T = 13.06\ldots$ or 13.1 or 13	A1 (3)
		(7 marks)

Question Number	Scheme		Marks	
Q3	$P = 80\mathrm{e}^{\frac{t}{5}}$			
(a)	$t = 0 \Rightarrow P = 80\mathrm{e}^{\frac{0}{5}} = 80(1) = \underline{80}$	$\underline{80}$	B1	
				(1)
(b)	$P = 1000 \Rightarrow 1000 = 80\mathrm{e}^{\frac{t}{5}} \Rightarrow \dfrac{1000}{80} = \mathrm{e}^{\frac{t}{5}}$	Substitutes $P = 1000$ and rearranges equation to make $\mathrm{e}^{\frac{t}{5}}$ the subject.	M1	
	$\therefore\ t = 5\ln\left(\dfrac{1000}{80}\right)$			
	$t = 12.6286...$	awrt 12.6 or 13 years	A1	
		Note $t = 12$ or $t = \text{awrt } 12.6 \Rightarrow t = 12$ will score A0		(2)
(c)	$\dfrac{\mathrm{d}P}{\mathrm{d}t} = 16\mathrm{e}^{\frac{t}{5}}$	$k\mathrm{e}^{\frac{1}{5}t}$ and $k \neq 80$.	M1	
		$16\mathrm{e}^{\frac{1}{5}t}$	A1	
				(2)
(d)	$50 = 16\mathrm{e}^{\frac{t}{5}}$			
	$\therefore\ t = 5\ln\left(\dfrac{50}{16}\right)$ $\{= 5.69717...\}$	Using $50 = \frac{\mathrm{d}P}{\mathrm{d}t}$ and an attempt to solve to find the value of t or $\frac{t}{5}$.	M1	
	$P = 80\mathrm{e}^{\frac{1}{5}\left(5\ln\left(\frac{50}{16}\right)\right)}$ or $P = 80\mathrm{e}^{\frac{1}{5}(5.69717...)}$	Substitutes their value of t back into the equation for P.	dM1	
	$P = \dfrac{80(50)}{16} = \underline{250}$	$\underline{250}$ or awrt 250	A1	
				(3)
				[8]

Question Number	Scheme	Marks
5 (a)	$p=7.5$	B1
		(1)
(b)	$2.5 = 7.5e^{-4k}$	M1
	$e^{-4k} = \frac{1}{3}$	M1
	$-4k = \ln(\frac{1}{3})$	dM1
	$-4k = -\ln(3)$	
	$k = \frac{1}{4}\ln(3)$	A1*
		(4)
(c)	$\frac{dm}{dt} = -kpe^{-kt}$ ft on their p and k	M1A1ft
	$-\frac{1}{4}ln3 \times 7.5e^{-\frac{1}{4}(ln3)t} = -0.6ln3$	
	$e^{-\frac{1}{4}(ln3)t} = \frac{2.4}{7.5} = (0.32)$	M1A1
	$-\frac{1}{4}(ln3)t = \ln(0.32)$	dM1
	t=4.1486…. 4.15 or awrt 4.1	A1
		(6)
		11Marks

Question Number	Scheme		Marks
4. **(a)**	$\theta = 20 + Ae^{-kt}$ (eqn *)		
	$\{t = 0, \theta = 90 \Rightarrow\}$ $90 = 20 + Ae^{-k(0)}$	Substitutes $t = 0$ and $\theta = 90$ into eqn *	M1
	$90 = 20 + A \Rightarrow \underline{A = 70}$	$\underline{A = 70}$	A1
			(2)
(b)	$\theta = 20 + 70e^{-kt}$		
	$\{t = 5, \theta = 55 \Rightarrow\}$ $55 = 20 + 70e^{-k(5)}$ $\frac{35}{70} = e^{-5k}$	Substitutes $t = 5$ and $\theta = 55$ into eqn * and rearranges eqn * to make $e^{\pm 5k}$ the subject.	M1
	$\ln\left(\frac{35}{70}\right) = -5k$	Takes 'lns' and proceeds to make '$\pm 5k$' the subject.	dM1
	$-5k = \ln\left(\frac{1}{2}\right)$		
	$-5k = \ln 1 - \ln 2 \Rightarrow -5k = -\ln 2 \Rightarrow \underline{k = \frac{1}{5}\ln 2}$	Convincing proof that $k = \frac{1}{5}\ln 2$	A1 *
			(3)
(c)	$\theta = 20 + 70e^{-\frac{1}{5}t\ln 2}$		
	$\frac{d\theta}{dt} = -\frac{1}{5}\ln 2.(70)e^{-\frac{1}{5}t\ln 2}$	$\pm \alpha e^{-kt}$ where $k = \frac{1}{5}\ln 2$ $-14\ln 2 e^{-\frac{1}{5}t\ln 2}$	M1 A1 oe
	When $t = 10$, $\frac{d\theta}{dt} = -14\ln 2 e^{-2\ln 2}$		
	$\frac{d\theta}{dt} = -\frac{7}{2}\ln 2 = -2.426015132...$		
	Rate of decrease of $\theta = 2.426\ ^{\circ}C/\text{min}$ (3 dp.)	awrt ± 2.426	A1
			(3) **[8]**