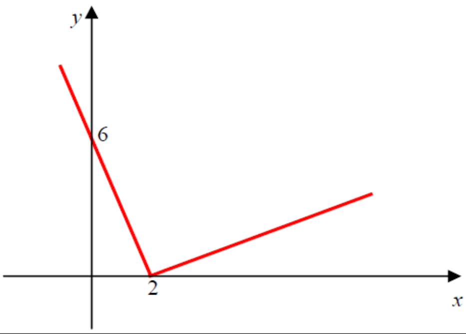
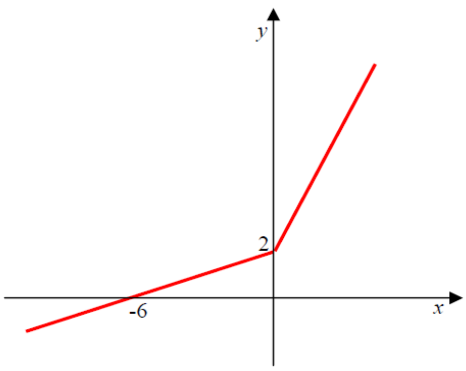
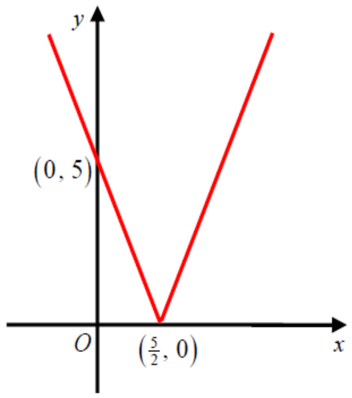
C3
Chapter 2
Functions

Question Number	Scheme	Marks
8.	(a) $x = 1 - 2y^3 \Rightarrow y = \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ *or* $\sqrt[3]{\frac{1-x}{2}}$	M1 A1 (2)
	$f^{-1}: x \mapsto \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ Ignore domain	
	(b) $gf(x) = \frac{3}{1-2x^3} - 4$	M1 A1
	$= \frac{3-4(1-2x^3)}{1-2x^3}$	M1
	$= \frac{8x^3-1}{1-2x^3}$ ∗ cso	A1 (4)
	$gf: x \mapsto \frac{8x^3-1}{1-2x^3}$ Ignore domain	
	(c) $8x^3 - 1 = 0$ Attempting solution of numerator = 0	M1
	$x = \frac{1}{2}$ Correct answer and no additional answers	A1 (2)
	(d) $\frac{dy}{dx} = \frac{(1-2x^3)\times 24x^2 + (8x^3-1)\times 6x^2}{(1-2x^3)^2}$	M1 A1
	$= \frac{18x^2}{(1-2x^3)^2}$	A1
	Solving their numerator = 0 and substituting to find y.	M1
	$x = 0,\ y = -1$	A1 (5) **[13]**

Question Number	Scheme	Marks
5 (a)	$\mathrm{g}(x) \geq 1$	B1 (1)
(b)	$\mathrm{f\,g}(x) = \mathrm{f}\left(\mathrm{e}^{x^2}\right) = 3\,\mathrm{e}^{x^2} + \ln \mathrm{e}^{x^2}$	M1
	$= x^2 + 3\,\mathrm{e}^{x^2}$ ✱	A1 (2)
	$\left(\mathrm{fg} : x \mapsto x^2 + 3\,\mathrm{e}^{x^2}\right)$	
(c)	$\mathrm{fg}(x) \geq 3$	B1 (1)
(d)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^2 + 3\,\mathrm{e}^{x^2}\right) = 2x + 6x\,\mathrm{e}^{x^2}$	M1 A1
	$2x + 6x\,\mathrm{e}^{x^2} = x^2\,\mathrm{e}^{x^2} + 2x$	
	$\mathrm{e}^{x^2}\left(6x - x^2\right) = 0$	M1
	$\mathrm{e}^{x^2} \neq 0$, $\quad 6x - x^2 = 0$	A1
	$x = 0, 6$	A1 A1 (6)
		[10]

Question Number	Scheme		Marks
6.			
(a)	$y = \dfrac{3-2x}{x-5} \Rightarrow y(x-5) = 3-2x$	Attempt to make x (or swapped y) the subject	M1
	$xy - 5y = 3 - 2x$		
	$\Rightarrow xy + 2x = 3 + 5y \Rightarrow x(y+2) = 3 + 5y$	Collect x terms together and factorise.	M1
	$\Rightarrow x = \dfrac{3+5y}{y+2} \quad \therefore \mathrm{f}^{-1}(x) = \dfrac{3+5x}{x+2}$	$\dfrac{3+5x}{x+2}$	A1 oe (3)
(b)	Range of g is $-9 \le \mathrm{g}(x) \le 4$ or $-9 \le y \le 4$	Correct Range	B1 (1)
(c)		Deduces that $\mathrm{g}(2)$ is 0. Seen or implied.	M1
	$\mathrm{g\,g}(2) = \mathrm{g}(0) = -6$, from sketch.	-6	A1 (2)
(d)	$\mathrm{fg}(8) = \mathrm{f}(4)$	Correct order g followed by f	M1
	$= \dfrac{3-4(2)}{4-5} = \dfrac{-5}{-1} = \underline{5}$	5	A1 (2)
(e)(i)		Correct shape 	B1
		$(2, \{0\}), (\{0\}, 6)$	B1
(e)(ii)		Correct shape 	B1
		Graph goes through $(\{0\}, 2)$ and $(-6, \{0\})$ which are marked.	B1 (4)
(f)	Domain of g^{-1} is $-9 \le x \le 4$	Either correct answer or a follow through from part (b) answer	B1 $\sqrt{}$ (1) **[13]**

4 (a)	$y = 4 - \ln(x+2)$ $\ln(x+2) = 4 - y$ $x + 2 = e^{4-y}$ $x = e^{4-y} - 2$ $f^{-1}(x) = e^{4-x} - 2$ oe	M1 M1A1 (3)
(b)	$x \leq 4$	B1 (1)
(c)	$fg(x) = 4 - \ln(e^{x^2} - 2 + 2)$ $fg(x) = 4 - x^2$	M1 dM1A1 (3)
(d)	$fg(x) \leq 4$	B1ft (1)
		8 Marks

Question Number	Scheme	Marks
4. (a)		M1A1
		(2)
(b)	$\underline{x = 20}$	B1
	$2x - 5 = -(15 + x)$; $\Rightarrow \underline{x = -\frac{10}{3}}$	M1;A1 oe.
		(3)
(c)	$\text{fg}(2) = \text{f}(-3) = \|2(-3) - 5\|; = \|-11\| = 11$	M1;A1
		(2)
(d)	$\text{g}(x) = x^2 - 4x + 1 = (x-2)^2 - 4 + 1 = (x-2)^2 - 3$. Hence $\text{g}_{\min} = -3$	M1
	Either $\text{g}_{\min} = -3$ or $\text{g}(x) \geqslant -3$ or $\text{g}(5) = 25 - 20 + 1 = 6$	B1
	$\underline{-3 \leqslant \text{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$	A1
		(3) **[10]**

(a) M1: V or  or  graph with vertex on the x-axis.

A1: $\left(\frac{5}{2}, \{0\}\right)$ and $\left(\{0\}, 5\right)$ seen and the graph appears in both the first and second quadrants.

(b) M1: Either $2x - 5 = -(15 + x)$ or $-(2x - 5) = 15 + x$

(c) M1: ***Full method*** of inserting $\text{g}(2)$ into $\text{f}(x) = \|2x - 5\|$ or for inserting $x = 2$ into $\left\|2(x^2 - 4x + 1) - 5\right\|$. There must be evidence of the modulus being applied.

(d) M1: **Full method** to establish the minimum of g. Eg: $(x \pm \alpha)^2 + \beta$ leading to $\text{g}_{\min} = \beta$. Or for candidate to differentiate the quadratic, set the result equal to zero, find x and insert this value of x back into $\text{f}(x)$ in order to find the minimum.

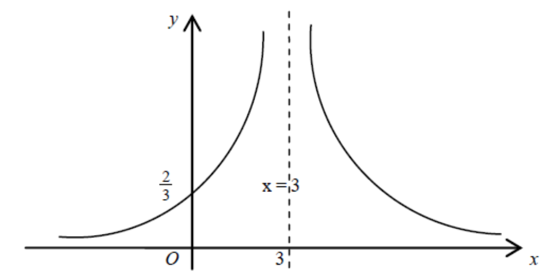
B1: For either finding the correct minimum value of g
(can be implied by $\text{g}(x) \geqslant -3$ or $\text{g}(x) > -3$) or for stating that $\text{g}(5) = 6$.

A1: $\underline{-3 \leqslant \text{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$ or $\underline{-3 \leqslant \text{g} \leqslant 6}$. **Note that:** $-3 \leqslant x \leqslant 6$ is A0.

Note that: $-3 \leqslant \text{f}(x) \leqslant 6$ is A0. **Note that**: $-3 \geqslant \text{g}(x) \geqslant 6$ is A0.

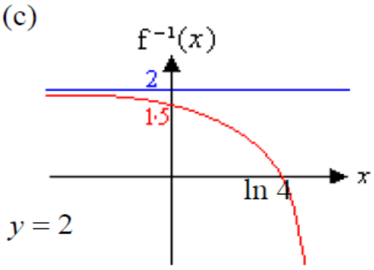
Note that: $\text{g}(x) \geqslant -3$ or $\text{g}(x) > -3$ or $x \geqslant -3$ or $x > -3$ with no working gains M1B1A0.

Note that for the final Accuracy Mark:
If a candidate writes down $-3 < \text{g}(x) < 6$ or $-3 < y < 6$, then award M1B1A0.
If, however, a candidate writes down $\text{g}(x) \geqslant -3$, $\text{g}(x) \leqslant 6$, then award A0.
If a candidate writes down $\text{g}(x) \geqslant -3$ or $\text{g}(x) \leqslant 6$, then award A0.

5. (*a*)	Finding g(4) = k and f(k) = …. or fg(x) = $\ln\left(\frac{4}{x-3}-1\right)$		M1	
	[f(2) = ln(2x2 – 1) fg(4) = ln(4 – 1)]	**=** ln 3	A1	(2)
(*b*)	$y = \ln(2x-1) \Rightarrow e^y = 2x-1$ or $e^x = 2y - 1$		M1, A1	
	$f^{-1}(x) = \frac{1}{2}(e^x+1)$ Allow $y = \frac{1}{2}(e^x+1)$		A1	
	Domain $x \in \Re$ [Allow $\Re$, all reals, $(-\infty, \infty)$]	independent	B1	(4)
(*c*)		Shape, and x-axis should appear to be asymptote	B1	
		Equation x = 3 needed, may see in diagram (ignore others)	B1 ind.	
		Intercept (0, $\frac{2}{3}$) no other; accept y = ⅔ (0.67) or on graph	B1 ind	(3)
(*d*)	$\frac{2}{x-3} = 3 \Rightarrow x = 3\frac{2}{3}$ or exact equiv.		B1	
	$\frac{2}{x-3} = -3$, $\Rightarrow x = 2\frac{1}{3}$ or exact equiv.		M1, A1	(3)
	Note: 2 = 3(x + 3) or 2 = 3(–x – 3) o.e. is M0A0			
Alt:	Squaring to quadratic ($9x^2 - 54x + 77 = 0$) **and** solving M1; B1A1			**(12 marks)**

Question Number	Scheme	Marks
4.	(a) $x^2-2x-3=(x-3)(x+1)$	B1
	$\text{f}(x)=\dfrac{2(x-1)-(x+1)}{(x-3)(x+1)}$ $\left(or\ \dfrac{2(x-1)}{(x-3)(x+1)}-\dfrac{x+1}{(x-3)(x+1)}\right)$	M1 A1
	$=\dfrac{x-3}{(x-3)(x+1)}=\dfrac{1}{x+1}$ * cso	A1 (4)
	(b) $\left(0,\dfrac{1}{4}\right)$ Accept $0<y<\dfrac{1}{4},\ 0<\text{f}(x)<\dfrac{1}{4}$ etc.	B1 B1 (2)
	(c) Let $y=\text{f}(x)$ $y=\dfrac{1}{x+1}$	
	$x=\dfrac{1}{y+1}$	
	$yx+x=1$	
	$y=\dfrac{1-x}{x}$ or $\dfrac{1}{x}-1$	M1 A1
	$\text{f}^{-1}(x)=\dfrac{1-x}{x}$	
	Domain of f^{-1} is $\left(0,\dfrac{1}{4}\right)$ ft their part (b)	B1 ft (3)
	(d) $\text{fg}(x)=\dfrac{1}{2x^2-3+1}$	
	$\dfrac{1}{2x^2-2}=\dfrac{1}{8}$	M1
	$x^2=5$	A1
	$x=\pm\sqrt{5}$ both	A1 (3)
		[12]

Question No		Scheme	Marks
7	(a)	$2x^2 + 7x - 4 = (2x-1)(x+4)$	B1
		$\dfrac{3(x+1)}{(2x-1)(x+4)} - \dfrac{1}{(x+4)} = \dfrac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$	M1
		$= \frac{x+4}{(2x-1)(x+4)}$	M1
		$= \frac{1}{2x-1}$	A1*
			(4)
	(b)	$y = \frac{1}{2x-1} \Rightarrow y(2x-1) = 1 \Rightarrow 2xy - y = 1$	
		$2xy = 1 + y \Rightarrow x = \dfrac{1+y}{2y}$	M1M1
		$y\ OR\ f^{-1}(x) = \dfrac{1+x}{2x}$	A1
			(3)
	(c)	x>0	B1
			(1)
	(d)	$\frac{1}{2\ln(x+1)-1} = \frac{1}{7}$	M1
		$\ln(x+1) = 4$	A1
		$x = e^4 - 1$	M1A1
			(4)
			12 Marks

Question Number	Scheme	Marks
6.	(a) $y = \ln(4-2x)$ $e^y = 4-2x$ leading to $x = 2-\frac{1}{2}e^y$ Changing subject and removing ln $y = 2-\frac{1}{2}e^x \Rightarrow f^{-1} \mapsto 2-\frac{1}{2}e^x$ ***** cso Domain of f^{-1} is □	M1 A1 A1 B1 **(4)**
	(b) Range of f^{-1} is $f^{-1}(x) < 2$ (and $f^{-1}(x) \in$ □)	B1 **(1)**
	(c)  Shape 1.5 ln 4	B1 B1 B1 B1 **(4)**
	(d) $x_1 \approx -0.3704$, $x_2 \approx -0.3452$ cao If more than 4 dp given in this part a maximum on one mark is lost. Penalise on the first occasion.	B1, B1 **(2)**
	(e) $x_3 = -0.354\,030\,19\ \ldots$ $x_4 = -0.350\,926\,88\ \ldots$ $x_5 = -0.352\,017\,61\ \ldots$ $x_6 = -0.351\,633\,86\ \ldots$ Calculating to at least x_6 to at least four dp $k \approx -0.352$ cao	M1 A1 **(2)** **[13]**
	Alternative to (e) $k \approx -0.352$ Found in any way Let $g(x) = x + \frac{1}{2}e^x$ $g(-0.3515) \approx +0.0003$, $g(-0.3525) \approx -0.001$ Change of sign (and continuity) $\Rightarrow k \in (-0.3525, -0.3515)$ $\Rightarrow k = -0.352$ (to 3 dp)	M1 A1 **(2)**