# Edexcel GCE

## Core Mathematics C3

**Advanced**

Thursday 17 January 2008 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

| Question Number | Leave Blank |
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| **Materials required for examination** | **Items included with question papers** |
| --- | --- |
| Mathematical Formulae (Green) | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions. Write your answers in the spaces provided in this question paper.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.
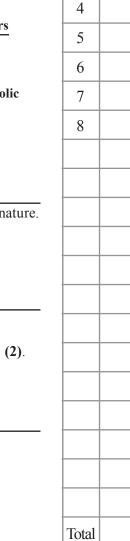
## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

*Turn over*

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1. Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)},$$

find the values of the constants $a$, $b$, $c$, $d$ and $e$.

(4)

**Question 1 continued**

**2.** A curve $C$ has equation

$$y = e^{2x} \tan x, \quad x \neq (2n+1)\frac{\pi}{2} .$$

(a) Show that the turning points on $C$ occur where $\tan x = -1$.

**(6)**

(b) Find an equation of the tangent to $C$ at the point where $x = 0$.

**(2)**

**Question 2 continued**

(Total 8 marks)

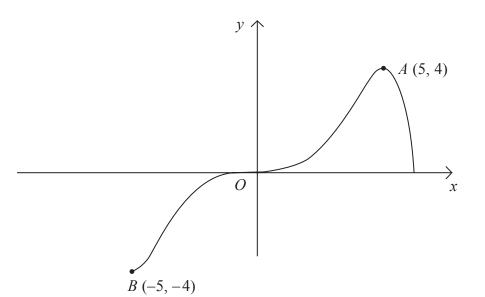**3.**  $$f(x) = \ln(x+2) - x + 1, \quad x > -2, x \in \mathbb{R}.$$

(a) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$.

**(2)**

(b) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \quad x_0 = 2.5$$

to calculate the values of $x_1, x_2$ and $x_3$ giving your answers to 5 decimal places.

**(3)**

(c) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places.

**(2)**

**Question 3 continued**

**4.**



$A$ (5, 4)

$O$

$x$

$y$

$B$ (−5, −4)

**Figure 1**

Figure 1 shows a sketch of the curve with equation $y = f(x)$.
The curve passes through the origin $O$ and the points $A(5, 4)$ and $B(-5, -4)$.

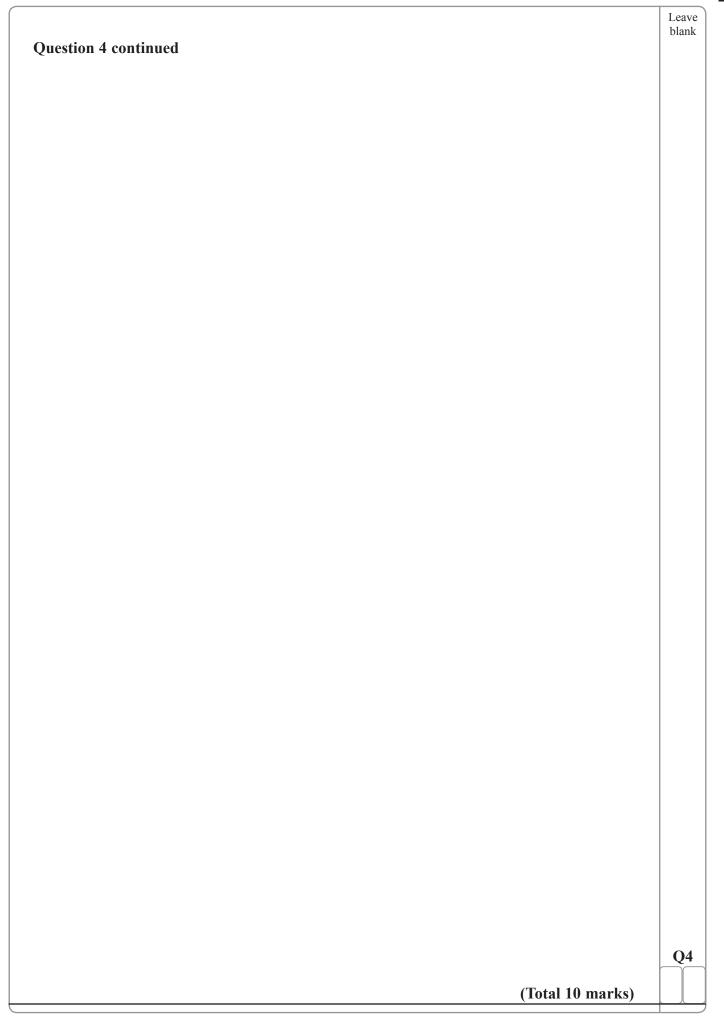In separate diagrams, sketch the graph with equation

(a)  $y = |f(x)|$,

**(3)**

(b)  $y = f(|x|)$,

**(3)**

(c)  $y = 2f(x+1)$.

**(4)**

On each sketch, show the coordinates of the points corresponding to $A$ and $B$.

**Question 4 continued**

H 2 6 3 1 5 R B 0 9 2 4

**Question 4 continued**

**Question 4 continued**

**Q4**

**(Total 10 marks)**

H 2 6 3 1 5 R B 0 1 1 2 4

**5.** The radioactive decay of a substance is given by

$$R = 1000e^{-ct}, \qquad t \geqslant 0.$$

where $R$ is the number of atoms at time $t$ years and $c$ is a positive constant.

(a) Find the number of atoms when the substance started to decay.

**(1)**

It takes 5730 years for half of the substance to decay.

(b) Find the value of $c$ to 3 significant figures.

**(4)**

(c) Calculate the number of atoms that will be left when $t = 22\ 920$ .

**(2)**

(d) In the space provided on page 13, sketch the graph of $R$ against $t$ .

**(2)**

**Question 5 continued**

**6.** (a) Use the double angle formulae and the identity

$$\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

**(4)**

(b) (i) Prove that

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \equiv 2\sec x, \qquad x \neq (2n+1)\frac{\pi}{2} \ .$$

**(4)**

(ii) Hence find, for $0 < x < 2\pi$ , all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4 \ .$$

**(3)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**Q6**

(Total 11 marks)

**Turn over**

7. A curve $C$ has equation

$$y = 3\sin 2x + 4\cos 2x, \quad -\pi \leqslant x \leqslant \pi .$$

The point $A(0, 4)$ lies on $C$.

(a) Find an equation of the normal to the curve $C$ at $A$.

(5)

(b) Express $y$ in the form $R\sin(2x+\alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

Give the value of $\alpha$ to 3 significant figures.

(4)

(c) Find the coordinates of the points of intersection of the curve $C$ with the $x$-axis. Give your answers to 2 decimal places.

(4)

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

(Total 13 marks)

Q7

**8.** The functions f and g are defined by

$$f : x \mapsto 1 - 2x^3, \ x \in \mathbb{R}$$

$$g : x \mapsto \frac{3}{x} - 4, \ x > 0, \ x \in \mathbb{R}$$

(a) Find the inverse function $f^{-1}$.

**(2)**

(b) Show that the composite function gf is

$$gf : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

**(4)**

(c) Solve $gf(x) = 0$.

**(2)**

(d) Use calculus to find the coordinates of the stationary point on the graph of $y = gf(x)$.

**(5)**

**Question 8 continued**

**Question 8 continued**

(Total 13 marks)

**TOTAL FOR PAPER: 75 MARKS**

**END**

**Q8**