C1 – PPQ's Sequences & Series

(1) $U_{n+1} = (U_n - 3)^2$

(a) $U_1 = 1$

$U_2 = (1-3)^2 = 4$
$U_3 = (4-3)^2 = 1$
$U_4 = (1-3)^2 = 4$

(b) $U_{20} = 4$

(2) $a_{n+1} = 3a_n - 5$

(a) $a_1 = 3$
$a_2 = 3(3) - 5 = 4$
$a_3 = 3(4) - 5 = 7$

(b) $a_4 = 3(7) - 5 = 16$
$a_5 = 3(16) - 5 = 43$
Sum $= 3 + 4 + 7 + 16 + 43 = \cancel{40}\ 73$

(b) ~~$a = 3,\ d = 3,\ n = 5$~~ ~~$S_5 = \frac{5}{2}[2(3) + (5-1)3] = \frac{5}{2}[6 + 12] = 45$~~

(3) $5 + 7 + 9 + \dots$

(a) $a = 5 \quad d = 2 \quad n = 200 \quad n^{th}$ term = ?

n^{th} term $= 5 + (200-1)2 = 403$p.

(b) $S_{200} = \frac{200}{2}[2(5) + (200-1)2] = 100(10 + 398) = \cancel{£40.80}$ £408.00.

(4) 11^{th} term $= 9$
$S_{11} = 77$
$n = 11$

(Sum) $77 = \frac{11}{2}[2a + (11-1)d]$

$154 = 11[2a + 10d]$
$154 = 22a + 110d \quad$ — (1)

(n^{th} term) $9 = a + (11-1)d$

$9 = a + 10d \quad$ — (2)

4 contd From 2 $a = 9 - 10d$ — 3

in 1 $154 = 22(9-10d) + 110d$

$154 = 198 - 220d + 110d$

$-44 = -110d$

$d = +0.4$

in 3 $a = 9 - 10(0.4) = 5$

5 (a) 11^{th} birthday $= 500$

12^{th} birthday $= 500 + 200 = 700$

So following 12^{th} birthday she has received $500 + 700 = £1200$

(b) $a = 500$, $d = 200$, $n = 8$

n^{th} term $= 500 + (8-1)200 = 500 + 1400 = £1900$

(c) $S = \frac{8}{2}[2(500) + (8-1)200] = 4[1000 + 1400] = £9600$

(d) $S_n = 32000$ $n = ?$

$32000 = \frac{n}{2}[2(500) + (n-1)200]$

$64000 = n[1000 + 200n - 200]$

$64000 = 800n + 200n^2$

$200n^2 + 800n - 64000 = 0$

$\div 200$

$n^2 + 4n - 320 = 0$

$(n+20)(n-16) = 0$

$n = 16$

$\therefore$ Alice is 26.

(6) $a_{n+1} = 3a_n + 5$

(a) $a_1 = k$

$a_2 = 3k+5$

(b) $a_3 = 3(3k+5)+5 = 9k+15+5 = 9k+20$ as required

(c) (i) $a_4 = 3(9k+20)+5 = 27k+60+5 = 27k+65$

$$\sum_{r=1}^{4} a_r = k + 3k+5 + 9k+20 + 27k+65$$

$$= 40k+90$$

(ii) $40k+90 = 10(4k+9)$ which is a multiple of 10

$\therefore \sum_{r=1}^{4} a_r$ will always be divisible by 10.

(7) (a) $S_n = a + (a+d) + (a+2d) + \dots + (a+(n-2)d) + (a+(n-1)d)$

reverse

$S_n = (a+(n-1)d) + (a+(n-2)d) + \dots + (a+d) + a$

add

$2S_n = [2a+(n-1)d] + [2a+(n-1)d] + \dots$ n times

$2S_n = n[2a+(n-1)d]$

$S_n = \frac{n}{2}[2a+(n-1)d]$ As required.

(b) 149, 147, 145, ...

$a=149$ $d=-2$ $n=21$

n^{th} term $= 149+(21-1)\times -2 = 149-40 = £109$

(c) $S_n = 5000$

$$5000 = \frac{n}{2}[2(149)+(n-1)\times -2]$$

$10000 = n[298-2n+2]$

$10000 = 300n - 2n^2$

$2n^2 - 300n + 10000 = 0$

÷2 $n^2 - 150n + 5000 = 0$ As required

(7)(d) $n^2 - 150n + 5000 = 0$

$(n-50)(n-100) = 0$

$n = 50$ or $n = 100$

(e) $n = 100$ is not a sensible solution, as the loan will have already been repaid after 50 months.

(8). $4 + 7 + 10 + \dots$

(a) $a = 4$ $d = 3$

n^{th} row $= 4 + (n-1)3 = 4 + 3n - 3 = 3n - 1$

(b) $S_{10} = \frac{10}{2}[2(4) + (10-1)3] = 5[8 + 27] = 175$ sticks.

(c) ~~$S_n = 1750$~~ k^{th} row uses fewer than 1750 sticks

$\therefore S_k < 1750$

$\frac{k}{2}[2(4) + (k-1)3] < 1750$

$k[8 + 3k - 3] < 3500$

$3k^2 + 5k - 3500 < 0$

$(3k - 100)(k + 35) < 0$ As required.

(d) $k > 0$ $\therefore k \neq -35$

$3k - 100 < 0$

$k < \frac{100}{3}$

$k < 33\frac{1}{3}$

$\therefore k = 33$.

9 $a = 30 \quad d = -1.5$

(a) 25^{th} term $= 30 + (25-1) \times -1.5$

$= 30 - 36$

$= -6$

(b) r^{th} term $= 0$

$30 + (r-1) \times -1.5 = 0$

$30 - 1.5r + 1.5 = 0$

$1.5r = 31.5$

$r = \frac{31.5}{1.5} = \frac{315}{15} = 21$

(c) $S_n > 0$

$\frac{n}{2}\left[2(30) + (n-1) \times -1.5\right] > 0$

$n\left[60 - 1.5n + 1.5\right]$

$n\left[61.5 - 1.5n\right] > 0$

either $n > 0$

or $61.5 - 1.5n > 0$

$-1.5n > -61.5$

$n < \frac{-61.5}{-1.5}$

$n < 41$

(c) S_{21} After 21^{st} term values become negative

So largest positive value of S_n will be S_{20}

$S_{20} = \frac{20}{2}\left[2(30) + (20-1) \times -1.5\right]$

$= 10\left(60 - 28.5\right]$

$= 315$.