C1 – JANUARY 2007

7(a) $f'(x) = 3x^2 - 6 - \frac{8}{x^2}$

$f'(x) = 3x^2 - 6 - 8x^{-2}$

$f(x) = \int 3x^2 - 6 - 8x^{-2}\, dx$

$f(x) = \frac{3}{3}x^3 - 6x - \frac{8}{-1}x^{-1} + c$

$f(x) = x^3 - 6x + \frac{8}{x} + c$

@ (2,1)

$1 = 2^3 - 6(2) + \frac{8}{2} + c$

$1 = 8 - 12 + 4 + c$

$c = 1 - 8 + 12 - 4 = 1$

$\therefore f(x) = x^3 - 6x + \frac{8}{x} + 1$

(b) @ P, gradient $f'(x) = 3(2)^2 - 6 - \frac{8}{2^2}$

$= 12 - 6 - 2$

$= 4$

$\therefore$ eqⁿ of tangent gradient 4 through (2,1)

Using $y - y_1 = m(x - x_1)$

$y - 1 = 4(x - 2)$

$y - 1 = 4x - 8$

$y = 4x - 7$

C1 - JANUARY 2008

Q9 (a) $f'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2}$

$f'(x) = 4x - 6x^{1/2} + 8x^{-2}$

$f(x) = \int 4x - 6x^{1/2} + 8x^{-2}\,dx$

$f(x) = \frac{4x^2}{2} - \frac{6}{3/2}x^{3/2} + \frac{8x^{-1}}{-1} + c$

$6 \div \frac{3}{2} = 6 \times \frac{2}{3} = \frac{12}{3} = 4$

$f(x) = 2x^2 - 4x^{3/2} - \frac{8}{x} + c$

@ (4,1)

$1 = 2(4)^2 - 4(\sqrt{4})^3 - \frac{8}{4} + c$

$1 = 32 - 32 - 2 + c$

$c = 3$

$\therefore f(x) = 2x^2 - 4x^{3/2} - \frac{8}{x} + 3$

(b) @ P, gradient $f'(x) = 4(4) - 6\sqrt{4} + \frac{8}{4^2}$

$= 16 - 12 + \frac{1}{2}$

$= 4\frac{1}{2} = \frac{9}{2}$

$\therefore$ gradient of normal $= -\frac{2}{9}$

through (4,1)

$y - 1 = -\frac{2}{9}(x-4)$

$\times 9$

$9y - 9 = -2x + 8$

$2x + 9y - 17 = 0$

C1 - May 2007

Q9 (a) $f'(x) = 6x^2 - 10x - 12$

$$f(x) = \int 6x^2 - 10x - 12 \, dx$$

$$f(x) = \frac{6}{3}x^3 - \frac{10}{2}x^2 - 12x + c$$

$$f(x) = 2x^3 - 5x^2 - 12x + c$$

@ (5, 65)

$$65 = 2(5)^3 - 5(5)^2 - 12(5) + c$$

$$65 = 250 - 125 - 60 + c$$

$$65 = 65 + c$$

$$c = 0$$

$$\therefore f(x) = 2x^3 - 5x^2 - 12x$$

(b) $f(x) = x(2x^2 - 5x - 12)$

$-24x$ $\quad -8x + 3x$

$$= x(2x^2 - 8x + 3x - 12)$$

$$= x(2x(x-4) + 3(x-4))$$

$$= x(2x+3)(x-4)$$ As required

(c) $y = x(2x+3)(x-4)$

crosses x axis when $y=0$: $x(2x+3)(x-4) = 0$

either $x = 0$ $\quad (0,0)$

or $2x+3=0$

$x = -3/2$ $\quad (-3/2, 0)$

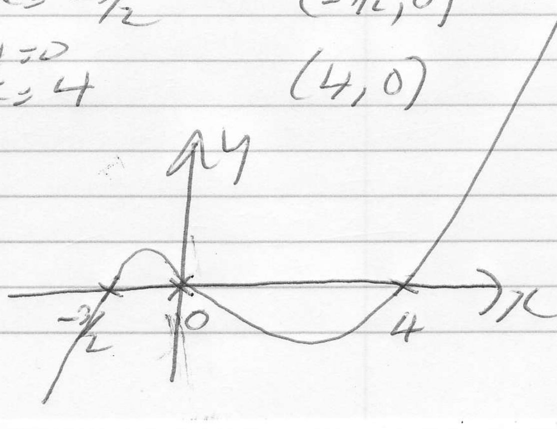
or $x - 4 = 0$

$x = 4$ $\quad (4,0)$

crosses y axis when $x=0$ $\quad y = 0$

as $x \to +\infty$ $\quad y = (+)(+)(+) = +$

as $x \to -\infty$ $\quad y = (-)(-)(-) = -$



C1 - May 2006

Q10(a) $f'(x) = 2x + \frac{3}{x^2}$

$f'(x) = 2x + 3x^{-2}$

$f(x) = \int 2x + 3x^{-2}\,dx$

$= \frac{2x^2}{2} + \frac{3x^{-1}}{-1} + c$

$f(x) = x^2 - \frac{3}{x} + c$

@ $(3, \frac{15}{2})$

$\frac{15}{2} = 3^2 - \frac{3}{3} + c$

$\frac{15}{2} = 9 - 1 + c$

$\frac{15}{2} - 8 = c$

$c = -\frac{1}{2}$

$\therefore\ f(x) = x^2 - \frac{3}{x} - \frac{1}{2}$

(b) $f(-2) = (-2)^2 - \frac{3}{-2} - \frac{1}{2} = 4 + \frac{3}{2} - \frac{1}{2} = 4 + 1 = 5$

(c) Gradient @ $(-2, 5)$ $f'(x) = 2(-2) + \frac{3}{(-2)^2} = -4 + \frac{3}{4} = -3\frac{1}{4}$

$= \frac{-13}{4}$

$\therefore$ eq^n of the line thru' $(-2, 5)$, gradient $\frac{13}{4}$

$y - 5 = \frac{-13}{4}(x - -2)$

$\times 4$

$4y - 20 = -13x - 26$

$13x + 4y + 6 = 0.$

C1 – May 2008

Q11 (a) $\frac{dy}{dx} = \frac{(x^2+3)^2}{x^2}$

$= \frac{(x^4+6x^2+9)}{x^2}$

$= x^{-2}(x^4+6x^2+9)$

$= x^2+6+9x^{-2}$ As required.

(b) $y = \int x^2+6+9x^{-2}\,dx$

$y = \frac{1}{3}x^3+6x+\frac{9}{-1}x^{-1}+c$

$y = \frac{1}{3}x^3+6x-\frac{9}{x}+c$

@ $(3,20)$

$20 = \frac{1}{3}(3)^3+6(3)-\frac{9}{3}+c$

$20 = 9+18-3+c$

$20 = 24+c$

$c = -4$

$\therefore\ y = \frac{1}{3}x^3+6x-\frac{9}{x}-4$