C1 – May 2007

Q3.

$y = 3x^2 + 4\sqrt{x}$

$y = 3x^2 + 4x^{\frac{1}{2}}$

(a) $\frac{dy}{dx} = 6x + 2x^{-\frac{1}{2}}$

(b) $\frac{d^2y}{dx^2} = 6 - x^{-\frac{3}{2}}$

(c) $\int 3x^2 + 4x^{\frac{1}{2}}\,dx = \frac{3}{3}x^3 + \frac{4}{\frac{3}{2}}x^{\frac{3}{2}} + c$

$= x^3 + \frac{8}{3}x^{\frac{3}{2}} + c$

C1 - May 2008

Q4 $f(x) = 3x + x^3$

(a) $f'(x) = 3 + 3x^2$

(b) $f'(x) = 15$

$3 + 3x^2 = 15$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$ but $x > 0$ $\therefore$ $x = 2$

C1 - JANUARY 2008

Q5(a) $\dfrac{2\sqrt{x}+3}{x} = x^{-1}(2x^{\frac{1}{2}}+3) = 2x^{-\frac{1}{2}}+3x^{-1}$

(b) $y = 5x - 7 + 2x^{-\frac{1}{2}} + 3x^{-1}$

$\dfrac{dy}{dx} = 5 - x^{-\frac{3}{2}} - 3x^{-2}$

C1 - May 2008

Q9 (a) $y = kx^3 - x^2 + x - 5$

$\frac{dy}{dx} = 3kx^2 - 2x + 1$

(b) equation $2y - 7x + 1 = 0$

rearrange to find gradient m

$$2y = 7x - 1$$
$$y = \frac{7x}{2} - \frac{1}{2}$$

$\therefore$ gradient of tangent @ A is $\frac{7}{2}$

$\therefore \frac{dy}{dx}$ @ $x = -\frac{1}{2}$ $= \frac{7}{2}$

$$\therefore \quad 3kx^2 - 2x + 1 = \frac{7}{2}$$

$$\cancel{3k\left(\frac{7}{2}\right)^2 - 2\left(\frac{7}{2}\right) =}$$

$$3k\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) = \frac{7}{2} - 1$$

$$\frac{3}{4}k + 1 = \frac{5}{2}$$

$$\frac{3}{4}k = \frac{3}{2}$$

$$k = \cancel{2}\,\frac{3}{2} \times \frac{4}{3} = 2$$

(c) $y = 2x^3 - x^2 + x - 5$

when $x = -\frac{1}{2}$

$$y = 2\left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 5$$

$$y = 2 \times -\frac{1}{8} - \frac{1}{4} - \frac{1}{2} - 5$$

$$y = -\frac{1}{4} - \frac{1}{4} - \frac{1}{2} - 5 = \cancel{\ } -6$$

C1 - May 2007

Q10 $y = x^2(x-6) + \frac{4}{x}$

(a) need coordinates of P and Q.

when $x=1$ $y = (1)^2(1-6) + \frac{4}{1}$

$= -5 + 4 = -1$ $\therefore P(1,-1)$

when $x=2$ $y = (2)^2(2-6) + \frac{4}{2}$

$= 4(-4) + 2$

$= -16 + 2$

$= -14$ $\therefore Q(2,-14)$

Now length of $PQ = \sqrt{(1-2)^2 + (-1 - -14)^2}$

$= \sqrt{(-1)^2 + (13)^2}$

$= \sqrt{1 + 169}$

$= \sqrt{170}$ As required

(b) If tangents are parallel, then their gradients are equal:

$y = x^2(x-6) + \frac{4}{x}$

$y = x^3 - 6x^2 + 4x^{-1}$

$\frac{dy}{dx} = 3x^2 - 12x - 4x^{-2} = 3x^2 - 12x - \frac{4}{x^2}$

when $x=1$ $\frac{dy}{dx} = 3(1)^2 - 12(1) - \frac{4}{(1)^2} = 3 - 12 - 4 = -13$

when $x=2$ $\frac{dy}{dx} = 3(2)^2 - 12(2) - \frac{4}{2^2} = 12 - 24 - 1 = -13$

$\therefore$ tangents are parallel

C1 - May 07

Q10 contd

(c) gradient of normal at $P(1,-1) = +\frac{1}{13}$

Using $y - y_1 = m(x - x_1)$

$$y - -1 = \frac{1}{13}(x-1)$$

$\times 13$

$$13y + 13 = x - 1$$

$$x - 13y - 14 = 0$$

C1 - JANUARY 2008

(10) $y = (x+3)(x-1)^2$

(a) when $x=0$ $\quad y = (0+3)(0-1)^2 = 3\times 1 = 3$ $\quad \therefore$ crosses y axis @ $(0,3)$

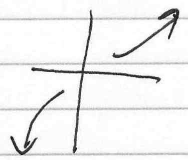
when $y=0$ $\quad (x+3)(x-1)^2 = 0$

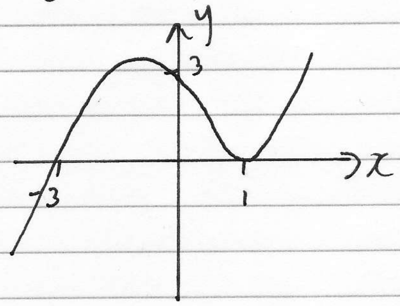
either $(x-1)^2 = 0$
$x = 1$ $\quad$ touches x axis @ $(1,0)$

or $x+3=0$
$x=-3$ $\quad$ crosses x axis @ $(-3,0)$

as $x \to +\infty$ $\quad y = (+)(+)^2 = +$

$x \to -\infty$ $\quad y = (-)(-)^2 = (-)(+) = -$





(b) $y = (x+3)(x^2-2x+1)$

$= x^3 - 2x^2 + x + 3x^2 - 6x + 3$

$= x^3 + x^2 - 5x + 3$ $\quad \therefore k = 3$

(c) $\frac{dy}{dx} = 3x^2 + 2x - 5$

when $\frac{dy}{dx} = 3$

$3x^2 + 2x - 5 = 3$

$3x^2 + 2x - 8 = 0$

$-24x$ $\quad +6x - 4x$

$3x^2 + 6x - 4x - 8 = 0$

$3x(x+2) - 4(x+2) = 0$

$(3x-4)(x+2) = 0$

$\therefore$ either $3x - 4 = 0$
$x = \frac{4}{3}$

or $x + 2 = 0$
$x = -2$