MARKING SCHEME

LEVEL 2 CERTIFICATE IN ADDITIONAL MATHEMATICS

SUMMER 2015

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2015 examination in LEVEL 2 CERTIFICATE IN ADDITIONAL MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

LEVEL 2 CERTIFICATE IN ADDITIONAL MATHEMATICS
Mark Scheme - Summer 2015

	Additional Mathematics Summer 2015		Final
1	$(2x - 5)(3x + 2)$ 5/2 and -2/3	B2 B2 4	B1 $(2x\ldots\ 5)(3x\ \ldots\ 2)$ or $(2x\ldots 2)(3x\ldots 5)$ Ignore sight of "=0" Must be from factorising, do not accept use of quadratic formula followed by 'factorising'. **MUST FT for their factors** FT for their factors equivalent difficulty not leading to whole number solutions. B1 for each answer Allow -0.66 or -0.7 as a solution provided $3x = -2$ is seen in working
2	(a) $(x +7)^2$ $(\pm\ \ldots)$ (Minimum value at x =) -7 (b) -40	M1 A1 B1 3	Ignore 'their $(\pm\ \ldots)$' or '=0' Do not accept method $dy/dx = 2x+14$ Unsupported -7 is M0, A0 CAO Must be given in (b), do not accept shown in (a). Do not accept reversed answers of '-40' in (a) and '-7' in (b), maximum mark would be possible M1 in (a) However, if no marks in (a), but a full statement, e.g. 'minimum value -40 when x = -7' is given in (b), then award B1 in (b) and M1, A0 in (a)
3	(a) $40x^7 - 6$ (+0) (b) $-8x^{-9}$ (or $-8/x^9$) (c) $2/5x^{-3/5}$ or equivalent	B3 B1 B1 5	B1 for $40x^7$ (not $5\times 8x^7$), B1 for -6 and B1 for +0 (or blank) provided at least one other mark awarded. CAO. Index needs to be simplified. CAO. Index needs to be simplified. ISW once simplified to stages shown in (b) and (c) Penalise including '+c' -1 only
4	(a) $(DE^2 =)\ (6- -4)^2 + (22 - 14)^2$ $(=10^2 + 8^2)$ $DE = \sqrt{164}$ $= 2\sqrt{41}$ (b) Gradient DE $(22 - 14) / (6 - -4)$ $= 8/10$ $(= 4/5)$ Gradient perpendicular -10/8 (= -5/4) (6+-4)/2, (22 + 14)/2 Mid point DE (1, 18) or equivalent Use of $y=mx+c$ or $y-y_1= m(x-x_1)$ $y = -10x/8 + 19¼$ or $y - 18 = -10/8\ (x - 1)$ $10x + 8y - 154=0$ OR $5x + 4y - 77 = 0$ OR $-10x - 8y + 154=0$ OR $-5x - 4y + 77 = 0$	M1 A1 B1 M1 A1 B1 M1 A1 M1 A1 A1 11	Or $(-4 - 6)^2 + (14 - 22)^2$. Allow 1 slip in sign of substitution CAO FT 'their DE' of equivalent difficulty expressed correctly, e.g. $\sqrt{18} = 3\sqrt{2}$, needs to be in the form $a\sqrt{b}$ where $a\neq 1$ and $b \neq 1$ or simpler Sight of $2\sqrt{41}$ implies previous $\sqrt{164}$ Or $(14 -22)/(-4 -6)$ CAO. Mark final answer and then FT FT -1/grad DE Accept $(1, \ldots)$ or $(\ldots , 18)$ CAO Method to find the equation using mid-point and perpendicular gradient (not 8/10) FT their mid-point (not D or E) & their perpendicular gradient, or FT substitution of their midpoint with their perpendicular gradient in $y = mx + c$ (towards finding c) *If no working for finding gradient is seen, then 'their 'spurious' incorrect perpendicular gradient' must be negative* FT for correct unsimplified form, not written in quotient form CAO. Form $ax + by + c = 0$ or a rearrange of this provided it is '=0'

	Additional Mathematics Summer 2015		Final
5	Sight of x-4 and x+1	B1	*FT 1 error in 1 side length, provided it is in the form 'x ± a' and not a single term expression, for M marks only*
	2(x(x-4) + x(x+1) + (x-4)(x+1)) (= 124)	M2	*Some work may be shown in stages, but implies the following M and m marks, but need to show summation* Correct expression for the total surface area Accept '×2' omitted if equated to 62 M1 for '×2' aspect omitted, or M1 for '×2' included with any 2 (of 3) area expressions correct, or M1 for '×2' included with any 4 (of 6) areas correct, or M1 if it could be correct but brackets missing and not implied in later working (if implied in later working allow M2)
	$2x^2 – 8x + 2x^2 + 2x + 2x^2 -8x + 2x – 8 = 124$ OR $x^2 – 4x + x^2 + x + x^2 -4x + x – 4 = 62$	m1	Equate to 124 with '×2' treated correctly, or equate to 62 if appropriate, with at least 2 of the 3 (or 4 of the 6) brackets expanded correctly; depends on M1
	$6x^2 – 12x – 8 = 124$ OR $3x^2 – 6x – 4 = 62$	A1	Depends on M2, m1 for correct manipulation and collection of terms
	Shows or sight of $x^2 –2x =22$ or $x^2 -2x - 22=0$	M1	Depends on M2, m1. Requirement to show as asked in the question Independent of other marks
	$(x – 1)^2 = 22 + 1$ OR $x = \frac{2 \pm\sqrt{92}}{2}$	A1	Must be correct to this stage of working FT for their quadratic equation of equivalent difficulty
	$x = 1 +\sqrt{23}$	QWC 2	CAO. Do not accept $1\pm\sqrt{23}$
	QWC2: • Candidates will be expected to present work clearly, with words explaining process or steps AND • make few if any mistakes in mathematical form, spelling, punctuation and grammar in their answer QWC1: Candidates will be expected to • present work clearly, with words explaining process or steps OR • make few if any mistakes in mathematical form, spelling, punctuation and grammar in their final answer		QWC2 Presents relevant material in a coherent and logical manner, using acceptable mathematical form, and with few if any errors in spelling, punctuation and grammar. QWC1 Presents relevant material in a coherent and logical manner but with some errors in use of mathematical form, spelling, punctuation or grammar OR evident weaknesses in organisation of material but using acceptable mathematical form, with few if any errors in spelling, punctuation and grammar. QWC0 Evident weaknesses in organisation of material, and errors in use of mathematical form, spelling, punctuation or grammar.
		10	

	Additional Mathematics Summer 2015		Final
6	$2x+1 = x^2 + 6x – 5$ $x^2 + 4x – 6 = 0$	M1 A1	Must be equated to zero. '=0' may be implied in further work to solve, if no further work and not '=0' then A0
	$x = \{-4 \pm\sqrt{(4^2 - 4\times1\times-6)}\}/2$	m1	FT provided their quadratic does not factorise and equivalent level of difficulty Use of correct quadratic formula, allow 1 slip in substitution (not a slip with the formula) If completing the square used award m1for sight of $(x + 2)^2 \pm \ldots$
	$x = \{-4 \pm\sqrt{40}\}/2$	A1	
	$x = 1.16$ and $x = -5.16$	A1	
	$x = 1.16$ with $y = 3.32$ or $y = 3.31$ **and** $x = -5.16$ with $y = -9.32$ or $y = -9.33$	A1	FT provided M1,m1 previously awarded using their values of x in 2x + 1 or equivalent to find y-values to 2 d.p.
		6	*Alternative using $x = (y – 1)/2$* M1 $y = (\underline{y-1})^2 + 6(\underline{y-1}) – 5$ or $y = y^2/4 + 5y/2 – 8 + ¼$ 2^2 2 A1 $y^2 + 6y – 31 = 0$ or equivalent (equate to zero) m1 $y = \{-6\pm\sqrt{(6^2 - 4\times1\times-31)}\}/2$ or equivalent (×¼) Allow 1 slip in substitution A1 $y = (-6\pm\sqrt{160})/2$ or equivalent A1 $y = 3.32$ and $y = -9.32$ A1 $x = 1.16$, $y = 3.32$ and $x = -5.16$, $y = -9.32$ FT to final A1, provided M1,m1 previously awarded using their values of y in $(y – 1)/2$ or equivalent to find x-values to 2 d.p.
7	(a) $3(-2)^3 - 2(-2)^2 + 5(-2) - 1$ (= -24-8-10-1) -43	M1 A1	Or division method giving $3x^2 - 8x \ldots$
	(b)(i) Substitute $x = 2$ Showing $f(2) = 0$	M1 A1	Or division method giving $x^2 + 10x \ldots$ Accept sight of substitution with '=0' shown
	(ii) $(x-2)(x^2 + bx + c)$ or intention to divide by $(x-2)$ with x^2 shown	M1	If any values are inserted at least 1 needs to be correct, appropriate sight of +10x or +21 implies M1 (and A1 to follow)
	$(x-2)(x^2 + 10x + 21)$	A2	A1 for (+)10x or (+)21 Or use of factor theorem A1 (x+3), A1 (x+7)
	$(x-2)(x+3)(x+7)$	A1 8	CAO, but ignore sight of "=0", ISW
8	(a) $180x^8$	B2	B1 for sight of $20x^9$. FT to 2nd B1 from $dy/dx = kx^n$ Ignore incorrect notation
	(b) $a = 1$ $c = 5$ $b = 3$	B1 B1 B1	FT $b = 8 –$ 'their c' or $b = 4 –$ 'their a'
		5	*Accept sight of correct answers from 'uncorrected' working* *Do not accept embedded answers, candidates need to identify values for a, b and c, not accept as left in working without clearly stating.*

	Additional Mathematics Summer 2015		Final
9	(a) $21x^7/7 - 3x^3/3 - x^{-1}/-1 + 6x$ $+ c$ (constant)	B4 B1	B1 for each term. Accept unsimplified $-x^{-1}/-1$. ISW Awarded if at least B1 for integration
	(b) $6x^3/3 + 4x^2/2$	B2	Mark final answer, then FT. B1 for sight of $6x^3/3$ or $4x^2/2$
	$[6x^3/3 + 4x^2/2]^5_2$	M1	FT their integration, not original. Intention to use 5, 2 **and** subtract
	$=(2\times5^3 + 2\times5^2) - (2\times2^3 + 2\times2^2)$	A1	FT for correct use of limits Accept unsimplifed fractions included
	$= 276$	A1 10	CAO, not FT. Do not accept '276 + c' *Answer only, no working shown M0 A0 A0*
10	(a) $(½)^2 + (\sqrt{3}/2)^2$ ¼ + ¾ (=1)	M1 A1	Use of ½ and $\sqrt{3}/2$ appropriately in either order
	(b) $5\times1 + 2\times\sqrt{3}/2 + \sqrt{3}$	M2	M1 for any 2 terms correct M1 only if first line of working is $5 + \sqrt{3} + \sqrt{3}$, but allow FT for possible A1
	$5 + 2\sqrt{3}$	A1 5	CAO. Mark final answer
11	3D visualisation with height 12cm AND use of appropriate methods	S1	**e.g.** 3D sketch with sight of height 12cm, Pythagoras' Theorem and tan ratio, or full alternative trig method following Pythagoras' Theorem, or use of base area to find the side length followed by full alternative trig method
	Side length of base 8cm or ½ side 4cm	B1	Sight of 8cm or 4cm as appropriate
	diagonal$^2 = 8^2 + 8^2$ OR (½ diagonal)$^2 = 4^2 + 4^2$	M1	FT their side length, provided ≠ 64, throughout other than the final A mark.
	diagonal is $\sqrt{128}$ or $8\sqrt{2}$ (cm) OR ½ diagonal is ½$\sqrt{128}$ or $4\sqrt{2}$	A1	(diagonal=11.3137…cm, ½diagonal = 5.65685…cm rounded or truncated)
	tan 'required angle' = (12 / ½diagonal) or **full** alternative method	M1	FT their derived ½ diagonal for M1 only, not 8 or 4(cm) Full alternative method: finding perpendicular height of a sloping face then the slant edge
	64.7(6…°) or 64.8(°)	A2 7	CAO. Accept 65(°) from correct working A1 for $\tan^{-1}(12/½\sqrt{128})$, or for an answer from premature approximation
12	$y+\delta y = (x+\delta x)^2 - 3(x+\delta x)$	B1	Or alternative notation. Allow if final bracket omitted
	Intention to subtract $(y=) x^2 - 3x$ to find δy	M1	
	$\delta y = 2x\delta x + (\delta x)^2 - 3\delta x$	A1	Accept δx^2 as meaning $(\delta x)^2$
	Dividing by δx **and** (lim) $\delta x \to 0$	M1	FT equivalent level of difficulty
	$dy/dx = \lim_{\delta x\to0} \delta y/\delta x = 2x - 3$	A1 5	CAO. Must follow from correct working *Use of dy/dx throughout or incorrect notation then possible maximum is only 4 marks, final A0*
13	When x = 3, finding y = -6	B1	
	$dy/dx = 4x - 8$	M1	
	when x = 3 gradient is 4	A1	
	Use of $y - y_1 = m(x - x_1)$ or $y = mx + c$	M1	Method to form equation FT their y value, but not y=3 and their derived gradient
	$y - -6 = 4(x-3)$ or $-6 = 4\times3 + c$, $c = -18$	A1	
	$4x - y - 18 = 0$	A1 6	CAO. Must be in this form, accept equivalents written as 3 terms with whole number coefficients with '=0' or '0='

	Additional Mathematics Summer 2015		Final
14	$(dy/dx=) 6x^2 - 24$ $dy/dx = 0$ or $6x^2 - 24 = 0$ $x = 2$ and $y = -19$ $x = -2$ and $y = 45$ $d^2y/dx^2 = 12x$ At (2, -19) $d^2y/dx^2 > 0$, point is a minimum At (-2, 45): $d^2y/dx^2 < 0$, point is a maximum	B1 M1 A1 A1 M1 A1 A1 7	 FT their dy/dx form $ax^2 + b$ throughout *Answer only, no working shown M0 A0 A0* *Method for determining min or max MUST be shown, final answer only is M0 here, then A0,A0* Or first derivative test, interpretation of first derivative test. Or alternative. FT for their x value FT for their other x value provided this does not have the same interpretation as the first x value *SC1 for correct FT from $d^2y/dx^2 = ax$, $a>0$* *Do not accept trial & improvement methods unless both stationary points are found correctly and confirmed as stated in the mark scheme*
15	(a) General cos curve intersecting x-axis only at (90°,0) and (270°,0) Correct curve with 5 and -5 on y-axis (b) 90° and 270° only	M1 A1 B1 3	Allow general shape as the joining of key values, but straight rather than clearly curving towards a turn at 0° and 360° in particular Must show a clear curve, not a straight a 0° and 360° in particular This values need to be selected, not amongst others unless unambiguous indicated as the response.
16	144	B1 1	*No marks if no working.* **Must** see 12^2 or '$(2\sqrt{3})^4 = 16\times9$'
17	(a) $(20)x^{8/8}/x^{2/3}$ or equivalent first stage of work evaluated correctly with simplification of indices $20x^{⅓}$ or $20\ ^3\sqrt{x}$ (b) **Correctly** extracting a factor of $x^{¼}$ (to give correct numerator) OR correct alternative method with one **correct** step towards simplification $2 + x^{½}$ or $2 + \sqrt{x}$	B1 B1 M1 A1 4	 CAO. Mark final answer For an alternative method, need sight of the two terms and $2 + \ldots$ or $\ldots + x^{2/4}$ for M1 CAO accepting $2 + x^{2/4}$. Mark final answer

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