Acceleration as a function of displacement

Consider if $a = f(x)$,

but $a = \frac{dv}{dt}$

So $\frac{dv}{dt} = f(x)$

use chain rule to introduce x...

$$\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$$

but $\frac{dx}{dt} = v$

$$\therefore \frac{dv}{dt} = v\frac{dv}{dx}$$

hence $a = v\frac{dv}{dx}$, $\quad v\frac{dv}{dx} = f(x) \Rightarrow \int v\,dv = \int f(x)\,dx$

$$* \; \frac{v^2}{2} = \int f(x)\,dx + C \; *$$

<u>Examples</u>

1. O and A are two points on a straight line with OA = d. A particle moves along the line with its acceleration a at any time t given by $a = -k^2x$, where x is the displacement of the particle from O at time t and k is a constant. When the particle is at A, its speed v is zero. Show that $v = k\sqrt{d^2 - x^2}$

2. OAB is a straight line with A between O and B and OA = 2m. A body is initially at rest at A and moves towards B with its acceleration at any instant given by $a = \frac{k}{x} ms^{-2}$, where x metres is the distance that the body is from O at that instant and k is a constant. Show that is v ms^{-1} is the velocity of the body, then $v^2 = 2k\ln\frac{x}{2}$.

3. If a = 4s + 2 and initially s = 0 and v = 1 ms^{-1}, find
    a. the value of v when s = 3m
    b. the value of t when s = 3m

<u>Exercise 1B Odd's</u>

Eg1 $a = -k^2 x$

$$\frac{V^2}{2} = \int -k^2 x \, dx$$

$$\frac{V^2}{2} = \frac{-k^2 x^2}{2} + c$$

when $x = d$, $v = 0$ $\therefore$ $c = \frac{k^2 d^2}{2}$

$$\therefore \frac{V^2}{2} = \frac{k^2 d^2}{2} - \frac{k^2 x^2}{2}$$

$$V^2 = k^2 (d^2 - x^2)$$

$V = \pm k\sqrt{d^2 - x^2}$ as x increases, a becomes more $-$ve but $\neq 0$ $\therefore V > 0$, $+$ve

$$V = k\sqrt{d^2 - x^2}$$

Eg2 $a = \frac{k}{x}$

$$\frac{V^2}{2} = \int \frac{k}{x} \, dx$$

$$\frac{V^2}{2} = k\ln x + c$$

when $V = 0$, $x = 2$ $\therefore$ $c = -k\ln 2$

$$\frac{V^2}{2} = k\ln x - k\ln 2$$

$$\frac{V^2}{2} = k\ln\left(\frac{x}{2}\right)$$

$V^2 = 2k\ln\left(\frac{x}{2}\right)$ As required.

Eg3 $a = 4s + 2$

$$\frac{V^2}{2} = \int 4s+2 \, ds$$

$$\frac{V^2}{2} = 2s^2 + 2s + c$$

when $s=0$ $V=1$ $\therefore c = \frac{1}{2}$

$$\frac{V^2}{2} = 2s^2 + 2s + \frac{1}{2}$$

$$V^2 = 4s^2 + 4s + 1$$

$$V^2 = (2s+1)^2$$

$V = \pm(2s+1)$ as a is increasing $V > 0$

$\therefore V = 2s+1$

(a) when $s=3$ $V = 7\,ms^{-1}$

(b) Now $V = \frac{ds}{dt}$

$$\therefore \frac{ds}{dt} = 2s+1$$

$$\int \frac{1}{2s+1} \, ds = \int dt$$

$$\frac{1}{2}\ln(2s+1) = t + c$$

when $t=0$, $s=0$ $\therefore c=0$

$$t = \frac{1}{2}\ln(2s+1)$$

when $s=3$

$$t = \frac{1}{2}\ln 7 = 0.973 \text{ secs.}$$