Further Kinematics

In Mechanics 1 acceleration was constant and this enabled us to use *uvast* ($v = u + at$, etc) formulae to model situations and solve problems.

In Mechanics 2 we had to deal with situations where the acceleration was variable and given as a function of time. These situations were modelled using calculus:

velocity = rate of change of distance

$$v = \frac{ds}{dt} = \dot{s}$$

The dot notation is used to show differentiation with respect to time.

acceleration = rate of change of velocity

$$a = \frac{dv}{dt} = \dot{v}$$

or

$$a = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2 s}{dt^2} = \ddot{s}$$

When the acceleration is given as a function of time, in order to obtain an expression for velocity, integration is necessary:

$$a = \frac{dv}{dt} = f(t)$$
$$\therefore \quad \int dv = \int f(t)dt$$
$$or \quad v = \int f(t)dt + C$$

The value of the constant needs to be determined using further information given in a question. A second integration will give an expression for displacement. *Remember that a constant must be introduced at each integration.*

Examples Review Exercise 1 Page 97 Questions 1, 4, 5 and 7.

Exercise 1A Q's 5, 8, 10 – 12.
When you're happy with those.....Q13 for megabeings!

Review Ex

(1) $a = 3e^{2t}$

$$a = \frac{dv}{dt} = 3e^{2t}$$

$$\int dv = \int 3e^{2t}\,dt$$

$$V = \frac{3}{2}e^{2t} + c$$

when $t=0$, $V=8$

$$8 = \frac{3}{2} + c \qquad c = \frac{13}{2}$$

$$V = \frac{dx}{dt} = \frac{3}{2}e^{2t} + \frac{13}{2}$$

$$x = \int \frac{3}{2}e^{2t} + \frac{13}{2}\,dt$$

$$x = \frac{3}{4}e^{2t} + \frac{13t}{2} + c$$

when $t=0$, $x_0 = \frac{3}{4} + c$

when $t=2$, $x_2 = \frac{3}{4}e^{4} + \frac{26}{2} + c$

$\therefore$ Distance moved in 1st 2 secs $= x_2 - x_0$

$$= \frac{3}{4}e^{4} + 13 + c - \left(\frac{3}{4} + c\right)$$

$$= \frac{3}{4}e^{4} + \frac{49}{4}$$

$$= 53.2\text{m}$$

(2) $V = 6t + t^2$

$$a = \frac{dv}{dt} = 6 + 2t$$

$$V = \frac{dx}{dt} = 6t + t^2$$

$$x = \int 6t + t^2 \, dt$$

$$x = 3t^2 + \frac{t^3}{3} + c$$

@ $t=2$ $\quad x_A = 3(2)^2 + \frac{(2)^3}{3} + c$

$$x_A = 12 + \frac{8}{3} + c$$

$$x_A = \frac{44}{3} + c$$

@ $t=5$ $\quad x_B = 3(5)^2 + \frac{(5)^3}{3} + c$

$$= 75 + \frac{125}{3} + c$$

$$= \frac{350}{3} + c$$

Dist of B from A $= x_B - x_A = \frac{350}{3} + c - \left(\frac{44}{3} + c\right) = 102$ metres

(4) $a = 6t - 8$

$a = \frac{dv}{dt} = 6t - 8$

$V = \int 6t - 8 \, dt$

$V = 3t^2 - 8t + c$

when $t = 0$, $V = 4$ $\therefore c = 4$

$V = 3t^2 - 8t + 4$ — (1)

$V = \frac{dx}{dt} = 3t^2 - 8t + 4$

$x = \int 3t^2 - 8t + 4 \, dt$

$x = t^3 - 4t^2 + 4t + c$

@ $t = 0$ $x = 0$ $\therefore c = 0$

$x = t^3 - 4t^2 + 4t$ — (2)

(a) Comes instantaneously to rest when $V = 0$

$\therefore$ from (1) $3t^2 - 8t + 4 = 0$

$t = \frac{2}{3}$ & $t = 2$

$\therefore$ first comes to rest @ $t = 2/3$

in (2) $x = \left(\frac{2}{3}\right)^3 - 4\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right)$

$= \frac{8}{27} - \frac{16}{9} + \frac{8}{3}$

$= \frac{32}{27}$ metres

(b) returns to starting point when $x = 0$

$t^3 - 4t^2 + 4t = 0$

$t(t^2 - 4t + 4) = 0$

$t(t-2)(t-2) = 0$

$\therefore$ either $t = 0$ $\Leftarrow$ sets off

or $t = 2$ secs $\Leftarrow$ returns

(4)(c) Max speed when $a=0$ $\quad 6t-8=0$

$$t=\frac{8}{6}=\frac{4}{3} \text{ sec}$$

in (1) $\quad V_{MAX}=3\left(\frac{4}{3}\right)^2-8\left(\frac{4}{3}\right)+4$

$$=\frac{48}{9}-\frac{32}{3}+4$$

$$=-\frac{4}{3}\ ms^{-1}$$

So for $0<t\leq 2$, $V_{max}=\frac{4}{3}\ ms^{-1}$

~~but for t<~~

but @ $t=0, V=4$ $\quad \therefore$ for $0\leq t\leq 2$, $V_{max}=4\ ms^{-1}$

(5)

$$a=kt$$

$$a=\frac{dv}{dt}=kt$$

$$V=\int kt\ dt$$

$$V=\frac{kt^2}{2}+c$$

when $t=0$ $V=4$ $\quad 4=c$

$$\therefore V=\frac{kt^2}{2}+4$$

when $t=10$ $V=12$ $\quad 12=50k+4$

$$k=\frac{8}{50}=\frac{4}{25}$$

$$\therefore V=\frac{2}{25}t^2+4$$

$$x=\int \frac{2}{25}t^2+4\ dt$$

$$x=\frac{2}{75}t^3+4t+c$$

$t=5$ $\quad x_5=\frac{2}{75}(5)^3+4(5)+c \quad =23\tfrac{1}{3}+c$

$t=10$ $\quad x_{10}=\frac{2}{75}(10)^3+4(10)+c \quad = 66\tfrac{2}{3}+c$

$$x_{10}-x_5=43\tfrac{1}{3} \text{ metres}$$

(7) $s = t + \sin 2t$

(a) $V = \frac{ds}{dt} = 1 + 2\cos 2t$

(b) $a = f = -4\sin 2t$

$f^2 = (-4\sin 2t)^2$

$f^2 = 16\sin^2 2t$

$f^2 = 16(1 - \cos^2 2t)$

From (a) $\frac{V-1}{2} = \cos 2t$

$f^2 = 16\left(1 - \frac{(V-1)^2}{4}\right)$

$f^2 = 16 - 4(V-1)^2$ As required